ASSIGNMENT 5

1 (a) $M = k \cdot \frac{1}{p}$, where k is a real number

(b) A function of the form $f(x) = mx + b$, where m and b are real numbers

(c) No, because of the $x^{-1} = \frac{1}{x}$ term

(d) When $b = 0$ (proportional = line through origin)

(e) linear: $y = ax + b, b \neq 0$ proportional: $y = ax$

ratio $\frac{\text{output}}{\text{input}}$ not constant ratio $\frac{\text{output}}{\text{input}}$ is constant

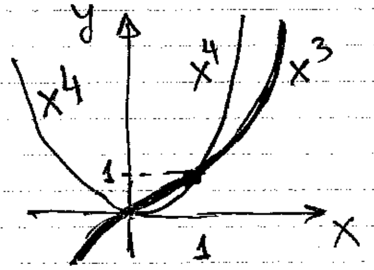
(f)

x	y
0	5
1	7
2	9
3	11
4	13

← slope is constant, but ratios $\frac{7}{1}, \frac{9}{2}, \frac{11}{3}, \frac{13}{4}$ are not equal

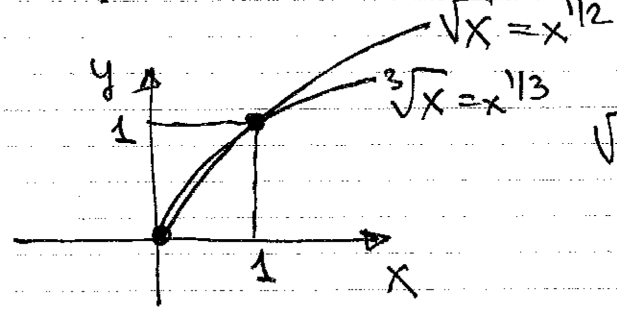
2 (a) $r < 0$ (graphs on page 52)

(b)



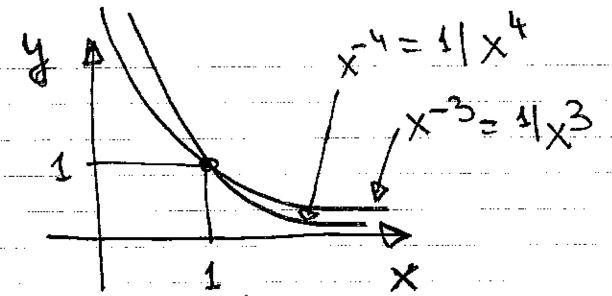
x^4 increases faster

(c)



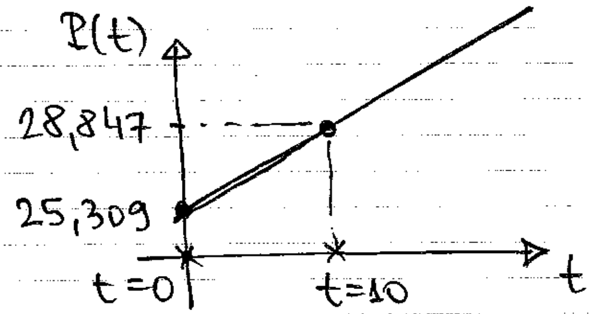
$\sqrt{x}$ increases faster

(d)



x^{-4} decreases faster

3. 1986 ... 25,309 ... t=0 (population in thousands)
1996 ... 28,847 ... t=10



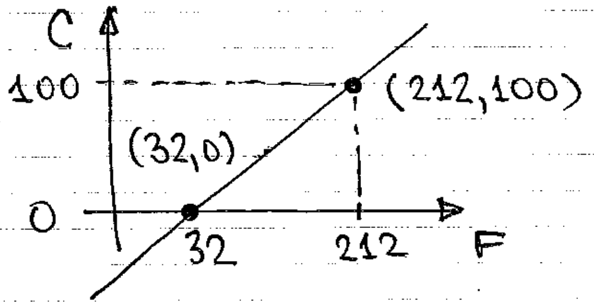
$$\text{slope} = \frac{28{,}847 - 25{,}309}{10} = 353.8$$

$$P(t) - 25{,}309 = 353.8(t-0)$$

so $P(t) = 25{,}309 + 353.8t$

prediction for 2006: $P(20) = 32{,}385$
(higher than actual population)

4. switch axes



$$\text{slope} = \frac{100-0}{212-32} = \frac{100}{180} = \frac{5}{9}$$

$$C - 0 = \frac{5}{9}(F-32)$$

$$C = \frac{5}{9}(F-32)$$

point-slope equation
using (32,0)

5. $$\text{BMI} = \frac{m}{h^2} \frac{[kg]}{[m^2]} = \frac{2.20426\,[lb]}{\left(\frac{100}{2.54}\right)^2 [in^2]}$$

$$= 0.0014221 \frac{[lb]}{[in^2]}$$

So if we want to have the same value in BMI numbers, then we need to multiply BMI by $1/0.0014221 = 703.18$

Thus: $$\text{BMI} = \frac{m}{h^2} \frac{[kg]}{[m^2]}$$

$$= 703.18 \cdot \frac{m}{h^2} \frac{[lb]}{[in^2]}$$

6. $$\text{BMI}_A = \frac{m}{h^2}$$

$$\text{BMI}_B = \frac{m}{(1.05h)^2} = \frac{1}{1.05^2} \cdot \frac{m}{h^2} = \underline{\underline{0.907}}\ \text{BMI}_A$$

$$\text{BMI}_C = \frac{0.95m}{h^2} = \underline{\underline{0.95}} \cdot \text{BMI}_A$$

So B has lower BMI

7. (a) $T(B) = a \cdot \sqrt[3]{B}$ so

$$T(2B) = a \cdot \sqrt[3]{2B} = \sqrt[3]{2} \cdot a\sqrt[3]{B} \approx 1.26\ T(B)$$

So if the body mass doubles, the blood circulation time increases by a factor of 1.26 (ie, by 26%)

(b) $T(B) = a \cdot \sqrt[3]{B} \rightarrow 152 = a\,\sqrt[3]{5400}$

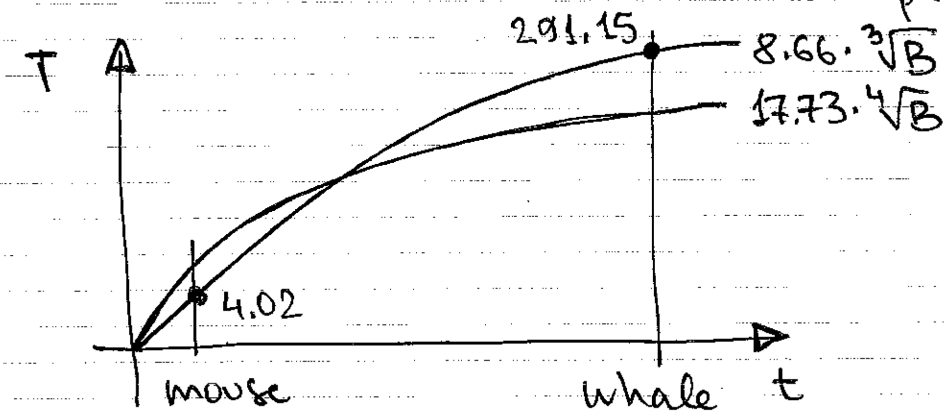
so $a = 152/\sqrt[3]{5400} \approx 8.66$

so $T(B) = 8.66 \cdot \sqrt[3]{B}$

(c) $T(0.1) = 8.66 \cdot \sqrt[3]{0.1} \approx 4.02\,s$ → smaller

$T(38{,}000) = 8.66 \cdot \sqrt[3]{38{,}000} \approx 291.15\,s$ → larger

compared to example 1.1.13



8. (a) Surface area is proportional to volume raised to the power of 2/3 $(S \propto V^{2/3})$

(b) When a baby grows to twice her size then the volume of her body (hence the mass) increases eight-fold. The strength of a bone is proportional to the cross-sectional area, and thus quadruples as the baby grows to twice her size. To compensate for the increase in mass, the bone thickness increases by more than a factor of 2 (precisely by a factor of 2.83)

(c) Radiocarbon dating can be used to date objects that are not older than about 57,000 years

(d) $C(t) = C(0)\, e^{kt}$ $\quad$ $C(0)$ = initial amount of ^{40}K

half life: $\quad 0.5\, \cancel{C(0)} = \cancel{C(0)}\, e^{k \cdot 1.248 \cdot 10^9}$

$$1.248 \cdot 10^9\, k = \ln 0.5$$

$$k = \frac{\ln 0.5}{1.248 \cdot 10^9} \approx -0.555406 \cdot 10^{-9}$$

so $\quad C(t) = C(0)\, e^{-0.555406 \cdot 10^{-9} \cdot t}$

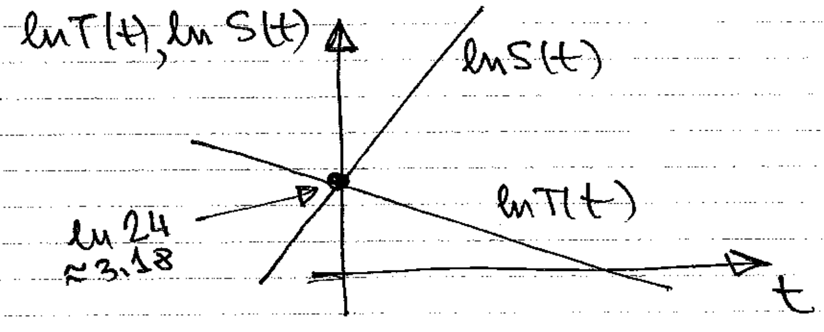
$$0.9645\, \cancel{C(0)} = \cancel{C(0)}\, e^{-0.555406 \cdot 10^{-9} t}$$

$$t = \frac{\ln(0.9645)}{-0.555406 \cdot 10^{-9}} \approx 6.5079 \cdot 10^7$$

$$\approx 65{,}079{,}000 \text{ years}$$

9. $\ln S(t) = \ln 24 + 1.8t \approx 1.8t + 3.18$

$\ln T(t) = \ln 24 - 0.8t \approx -0.8t + 3.18$



(negative t might, or might not make sense, depending on context)

10. min = -1 $\quad$ amplitude = 5

max = 9 $\quad$ period = 2

average = 4 $\quad$ phase = 0

11.(a) $y = \sin\left(4\left(t+\frac{\pi}{4}\right)\right)$ ... sine graph compressed by a factor of 4 then shifted $\pi/4$ units to the left

min = −1
max = 1
average = 0
amplitude = 1

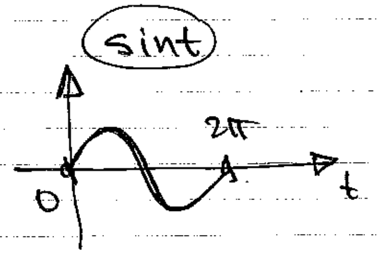
period $= \frac{2\pi}{4} = \frac{\pi}{2}$
shift $\frac{\pi}{4}$ to the left (or: phase $= -\frac{\pi}{4}$)

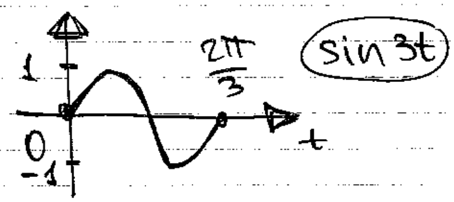
(b) $y = \cos\left(\frac{t}{2}\right) + 5$ ... cosine graph stretch by a factor of 2 then up 5 units

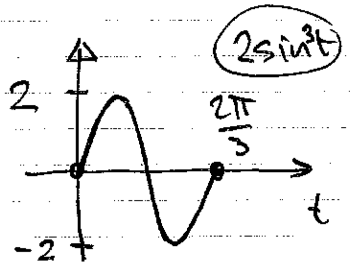
min = 4
max = 6
average = 5
amplitude = 1

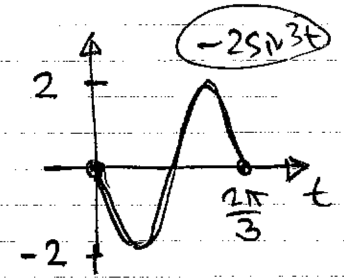
period $= 2\pi/_{1/2} = 4\pi$
shift (phase) = 0

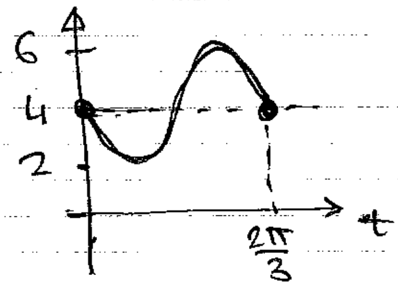
(c) $y = -2\sin(3t)+4$ ... sine graph, compressed horizontally by a factor of 3, expanded vertically by a factor of 2, reflected across x-axis, moved up 4 units



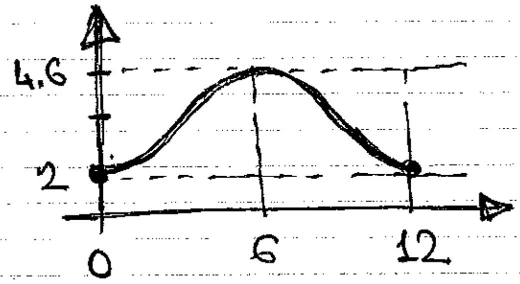








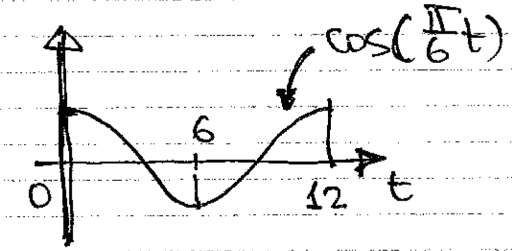
min = 2, max = 6, average = 4, amplitude = 2
phase = 0, period $= 2\pi/3$

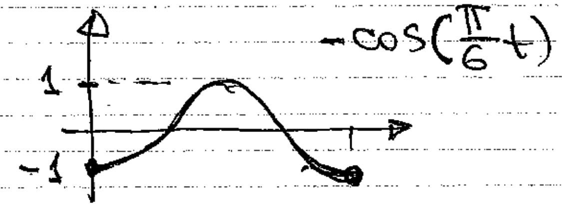
12. 

period is 12

$$\frac{2\pi}{a}=12 \to a=\frac{2\pi}{12}=\frac{\pi}{6}$$

use cosine: $\cos(at) \to \cos\left(\frac{\pi}{6}t\right)$





(average is $\frac{2+4.6}{2}=3.3$
amplitude is 1,3)

$$\boxed{-1{,}3\cos\left(\frac{\pi}{6}t\right)+3.3}$$