1A03 – CALCULUS I FOR SCIENCE
(SECTION C02)

Lecture 18

Today

CURVE SKETCHING

– Summarizing our knowledge about graph shape

Checklist

(A) Domain – where is $f(x)$ defined?

(B) Intercepts : x with $f(x)=0$ and $f(0)$ – identify these if easy

(C) Symmetry : $f(x)$ – even $f(-x) = f(x)$
– odd $f(-x) = -f(x)$

$f(x)$ periodic $f(x+T) = f(x)$ (T ← period) (what is T? It is suggested by form of $f(x)$ e.g. contains $\cos(3x)$ – perhaps $T = \frac{2\pi}{3}$?)

(D) Asymptotes

– behaviour at $\pm\infty$ (do we have $\lim_{x\to\pm\infty} f(x) = \#$ or $\pm\infty$?) (# ← HA)

– are there points a for which $\lim_{x\to a} f(x) = \pm\infty$? (← VA)
(points a come from infinite discontinuities)

(E) Increasing/Decreasing – where is $f'(x) = 0$? Then $\gtrless 0$?

(F) Local Max./Min. – where is $f'(x)$ undefined? (i.e. find all critical #s) Then use 1st / 2nd derivative test

(To have a local max./min at c, $f(x)$ must be defined at $x=c$.)

Find $f''(x)$

(G) Concavity – where is $f''(x)=0$? Then $\gtrless 0$?
& where are points of inflection?

where this changes & $f(x)$ defined

<u>Example</u> Sketch the graph of $f(x) = \dfrac{2x}{x^2+3}$.

<u>Solution</u> (A) Defined everywhere (x^2+3 has no real roots!)

(B) $f(x)=0$ when $2x=0$ i.e. $x=0$

$f(0)=0$ so only intercept is $(0,0)$.

(C) $f(-x) = \dfrac{2(-x)}{(-x)^2+3} = -\dfrac{2x}{x^2+3} = -f(x)$ odd

Nothing about $f(x) = \dfrac{2x}{x^2+3}$ suggests periodicity

(D) $f(x)$ continuous everywhere (rational function)
so no VA

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{2x}{x^2+3} \underset{\substack{\uparrow \\ L'H\ \frac{\infty}{\infty}}}{=} \lim_{x\to\infty} \frac{\cancel{2}\,1}{\cancel{2}x} = 0.$$

As for $x\to-\infty$, either check the same way or use $f(x)$ odd

– we get $\lim\limits_{x\to-\infty} f(x) = 0$. HA $y=0$ at $\pm\infty$.

(E) $f'(x) = 0$? . $f'(x) = \dfrac{2(x^2+3) - (2x)(2x)}{(x^2+3)^2}$

$$= \frac{-2x^2+6}{(x^2+3)^2} = \frac{-2(x^2-3)}{(x^2+3)^2}$$

So $f'(x) = 0$ when $x = \pm\sqrt{3}$

Note $(x^2+3)^2 > 0$

Check : $(-\infty, -\sqrt{3}) \longrightarrow f'(x) < 0$ dec.

$(-\sqrt{3}, \sqrt{3}) \longrightarrow f'(x) > 0$ inc.

$(\sqrt{3}, \infty) \longrightarrow f'(x) < 0$ dec.

(F) At $x = -\sqrt{3}$, $f'(x)$ dec. $\to$ inc. so local min.

At $x = +\sqrt{3}$, $f'(x)$ inc. $\to$ dec. so local max.

(G) $f''(x) = 0$? $f''(x) = \underset{\text{Check!}}{\dots} = \dfrac{4x(x^2-9)}{(x^2+3)^3}$

+ve

So $f''(x) = 0$ when $x = 0$ and $x = \pm 3$

Check : on $(-\infty, -3)$, $f''(x) < 0$ CD

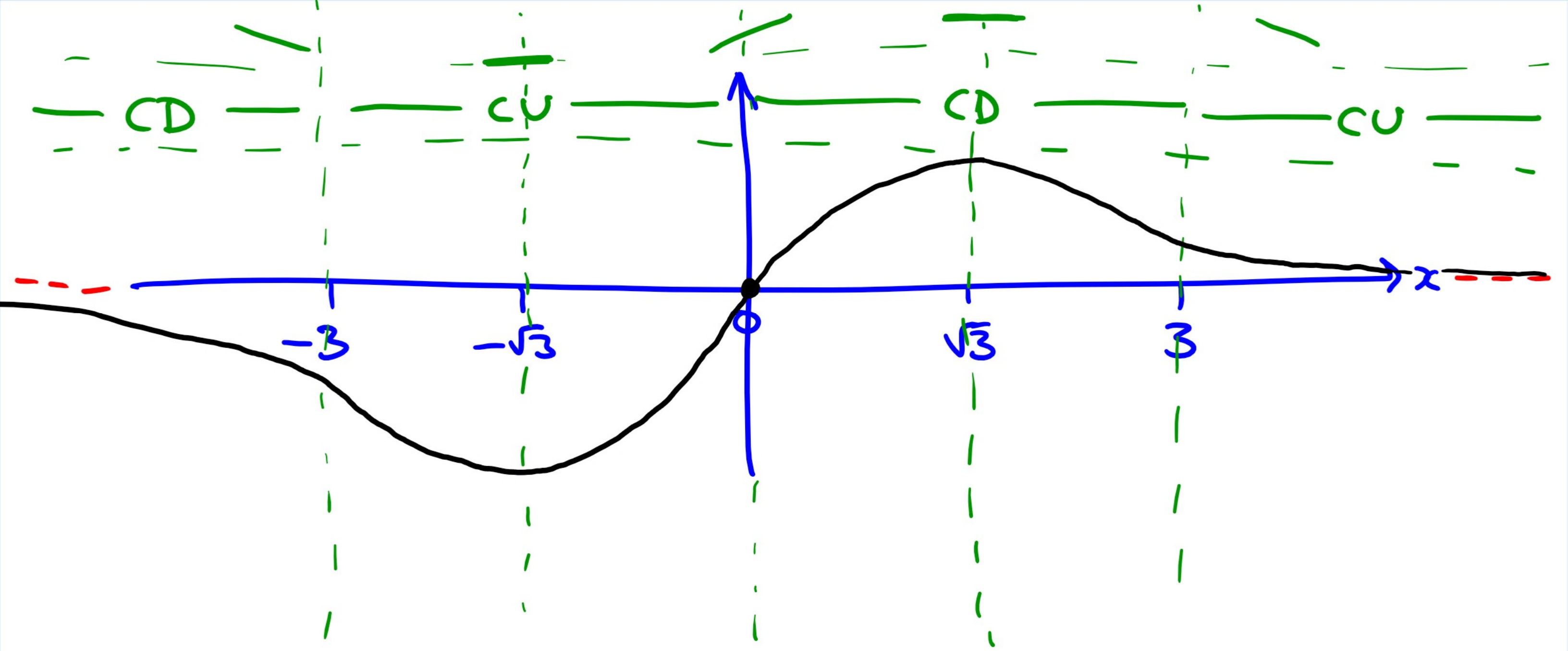
on $(-3, 0)$, $f''(x) > 0$ CU

on $(0, 3)$, $f''(x) < 0$ CD

on $(3, \infty)$, $f''(x) > 0$ CU

Each pt $-3, 0, 3$ is a point of inflection

Finally Draw the graph!



Exercise / Example Sketch the graph of
$$f(x) = 2xe^{1/x}.$$

Solution (A) Defined for $x \neq 0$

(B) $f(0)$ not defined

$f(x) = 0$ never happens ($e^{1/x} > 0$ as $e^{\sim} > 0$ always & $f(0)$ not defined)

(C) $f(-x) = 2(-x)e^{1/(-x)}$

$$= -2xe^{-1/x} \neq \begin{cases} -2xe^{1/x} & \text{not odd} \\ 2xe^{1/x} & \text{not even} \end{cases}$$

... Exercise.