1A03 — CALCULUS I FOR SCIENCE
(SECTION C02)

Lecture 19

Last time Curve Sketching

(A) Domain
(B) Intercepts
(C) Symmetry (odd, even, periodic)
(D) Asymptotes (Limits to/at $\pm\infty$)
(E) Increasing / Decreasing ($f'(x) \gtrless 0$)
(F) Local Max./Min. ($f'(x)=0$/DNE & 1st/2nd deriv. test)
(G) Concavity ($f''(x) \gtrless 0$) & points of inflection.

Exercise Sketch $f(x) = 2xe^{1/x}$.

(D) HA: $\displaystyle\lim_{x\to\infty} \underbrace{2x}_{\to\infty}\, e^{\overbrace{1/x}^{\to \frac{1}{\infty}=0}} = \infty \cdot 1 = \infty$

$\displaystyle\lim_{x\to-\infty} \underbrace{2x}_{\to -\infty}\, \underbrace{e^{1/x}}_{\to 1} = -\infty$

VA: Check points of discontinuity (VA happens at infinite discontinuities)

Here that just means $x=0$ (f undefined)

Check both $\displaystyle\lim_{x\to0^+} f(x)$ and $\displaystyle\lim_{x\to0^-} f(x)$

$\displaystyle\lim_{x\to0^+} \underbrace{2x}_{\to 0}\, \underbrace{e^{1/x}}_{\to\infty} = \underbrace{\ldots}_{\text{L'H } 0\cdot\infty} = \infty$ (Check!)

$\displaystyle\lim_{x\to0^-} \underbrace{2x}_{\to 0}\, \underbrace{e^{1/x}}_{\to 0} = 0 \cdot 0 = 0$

← VA only on one side of $x=0$.

To continue with (E), (F), (G), remember, $x=0$ is a point of interest, where $f'(x)$ and $f''(x)$ are NOT defined.

(E) $f'(x)$ has a zero at $x=1$ (Check this by finding $f'(x)$!)

So critical #s are $x=0$ and $x=1$

It turns out that $f'(x) > 0$ on $(-\infty, 0)$ - inc.

< 0 on $(0,1)$ - dec.

> 0 on $(1, \infty)$ - inc.

Check this by finding where $f'(x) \gtrless 0$!

(F) $x=0$ not local max./min. as f not defined there (even though $f'(x)$ changes sign there)

but we see $x=1$ is a local minimum. (changes from dec. to inc.)

(G) $f''(x)=0$ has no solutions

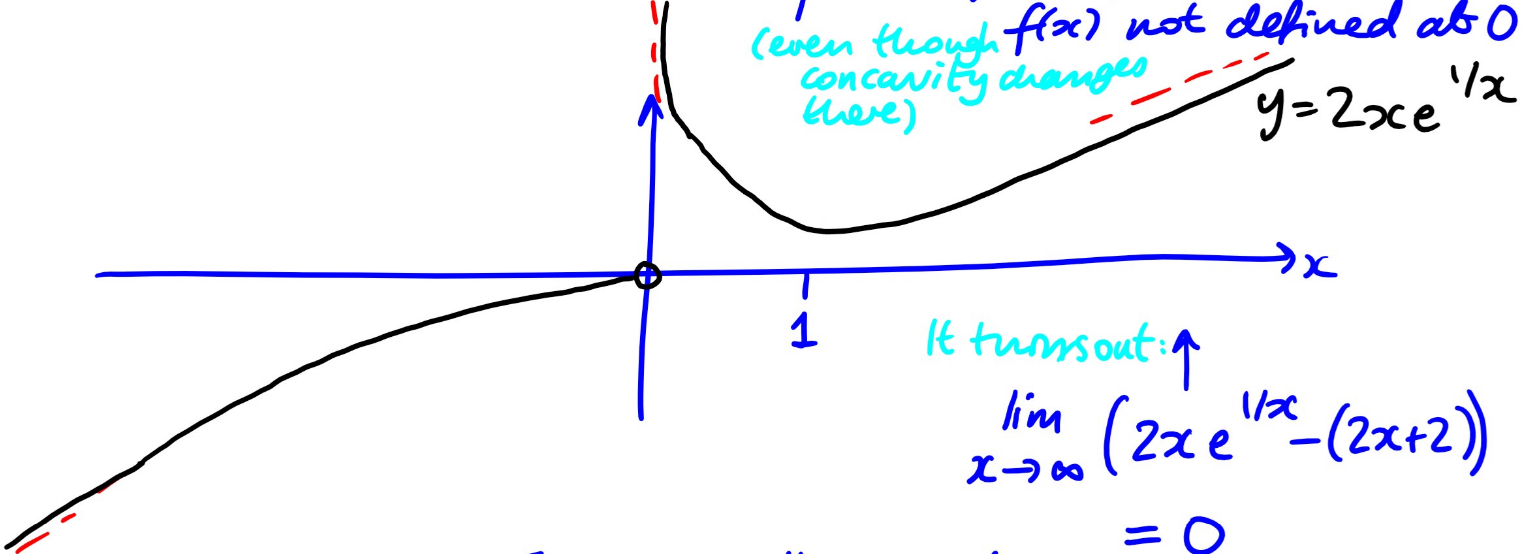
So check $f''(x)$ on $(-\infty, 0)$, $(0, \infty)$ ← i.e. still need to care about $x=0$ where $f''(x)$ undefined

$f''(x)<0$ $\quad$ $f''(x)>0$

CD $\quad$ CU

↑

0 not point of inflection as $f(x)$ not defined at 0 (even though concavity changes there)



It turns out: $\lim_{x\to\infty}\left(2xe^{1/x} - (2x+2)\right) = 0$

This is called a <u>slant asymptote</u>.

In general, $g(x)$ has a <u>slant asymptote</u> at ∞ (or $-\infty$)

if there is a line $y = mx + c$ with $\lim_{x \to \infty} (g(x) - (mx + c)) = 0$ (or $x \to -\infty$)

Oblique asymptote: ambiguous as to whether this must be a <u>line</u> or not - just not VA or HA.

4.7 Optimization

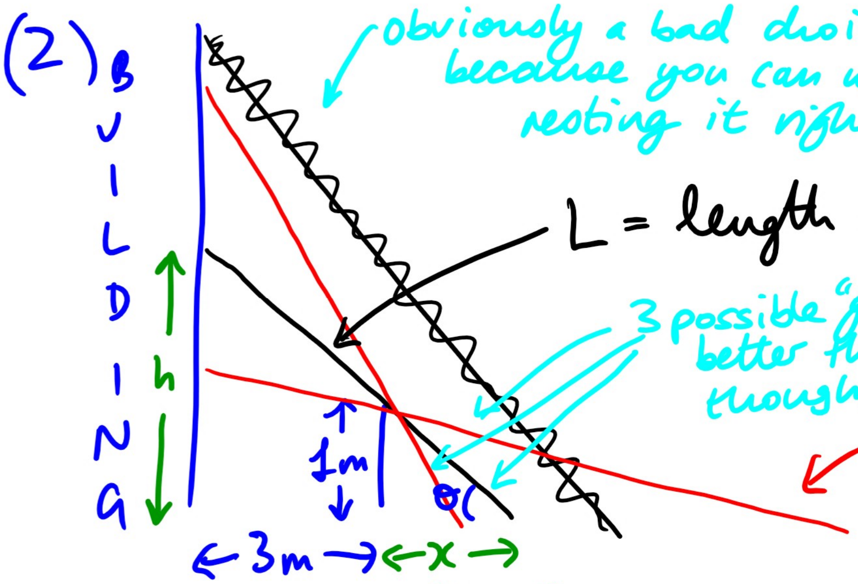
- Practical application of finding absolute maximum/minimum
- Want to maximise/minimise some function (may represent profits, costs, travel time...)
  (max.) (min.) (min.)
- Need to translate words problems into equations

<u>Example</u> A 1m high fence runs alongside a building at a distance of 3m away. What is the length of the shortest ladder to clear the fence & lean against the building?

<u>Solution</u> (1) Understand the problem:

- unknowns - length of ladder
- given quantities - height of fence, distance of fence from building
- constraints (conditions/restrictions) - ladder must clear fence

(2)



obviously a bad choice (once you see a picture) because you can use a shorter ladder by resting it right on top of the fence.

L = length of ladder

3 possible "good" choices, but some clearly better than others; in particular, though, the Picture tells us:

There will be a minimum solution

Introduce new variables for unknown quantities & write down equations relating them:

e.g. $L^2 = h^2 + (x+3)^2$ and $\frac{h}{3+x} = \frac{1}{x}$ $(= \tan\theta)$

or similar Δs (congruence)

Goal: express L as a function of $\underline{\underline{\text{one}}}$ variable

the quantity you want to minimise

Here we can rearrange

$$h = \frac{3+x}{x}$$

& substitute:
$$L^2 = \left(\frac{3+x}{x}\right)^2 + (x+3)^2$$
$$L^2 = (x+3)^2\left(\frac{1}{x^2} + 1\right)$$

Minimise this! <u>TBC</u>