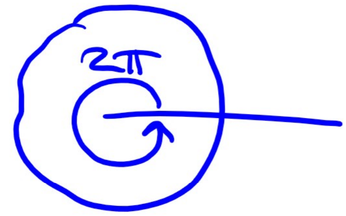
1A03 – CALCULUS I FOR SCIENCE
(SECTION C02) Lecture 2

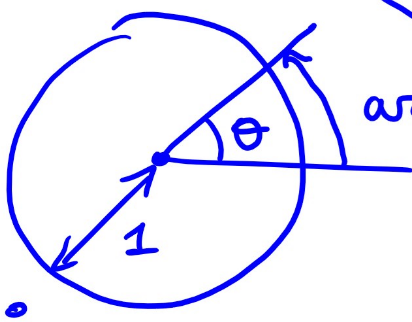
TRIGONOMETRY

From now on: radians

2π of them in a circle



In a unit circle



arc length = θ

i.e. 2π rad $= 360^\circ$

To go from rad ⟶ deg: divide by 2π & multiply by 360

i.e. mult. by $\frac{360}{2\pi} = \frac{180}{\pi}$

& deg ⟶ rad: mult. by $\pi/180$.

Examples Find 45° in radians.

Solution $45 \times \pi/180 = \pi/4$ rad.

Examples Find $\pi/3$ in degrees.

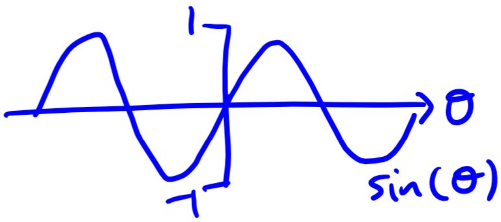
Solution $\pi/3 \times 180/\pi = 60^\circ$.

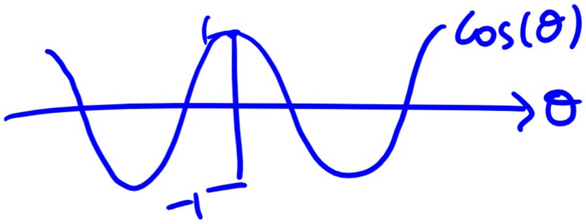
Trig. functions

sin , cos , tan

csc , sec , cot

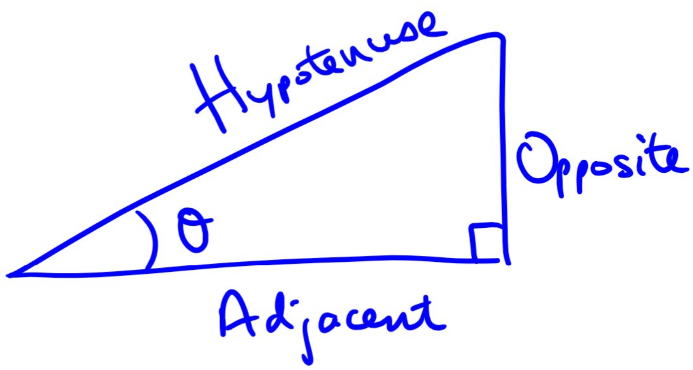
$= \frac{1}{\sin}$ $= \frac{1}{\cos}$ $= \frac{1}{\tan}$





How to define these?

If θ is acute:



SOH CAH TOA

Students of Hamilton

Care About ...

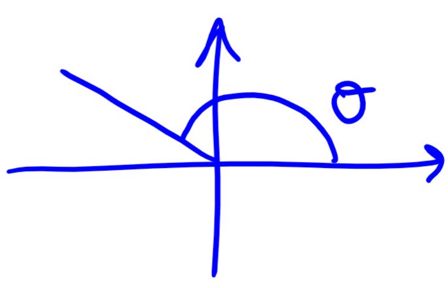
↑ Send me your suggestions!!

$\sin\theta = O/H$ $\csc\theta = H/O$

$\cos\theta = A/H$ $\sec\theta = H/A$

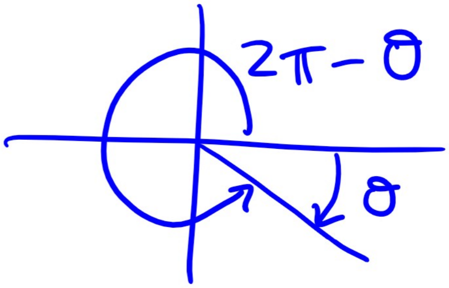
$\tan\theta = O/A$ $\cot\theta = A/O$

If θ (not) acute, we look at θ in standard position:

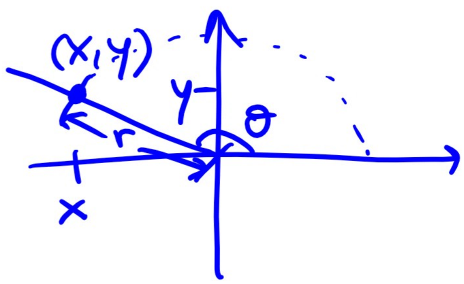


Start at x-axis and rotate counter-clockwise through angle θ

Note we rotate clockwise if θ is negative.



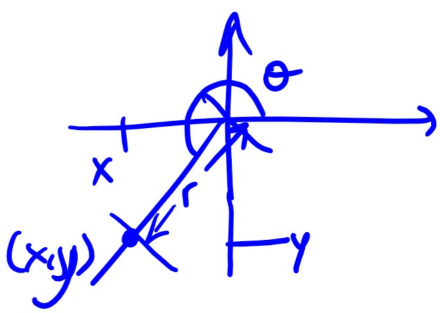
Trig. functions in general



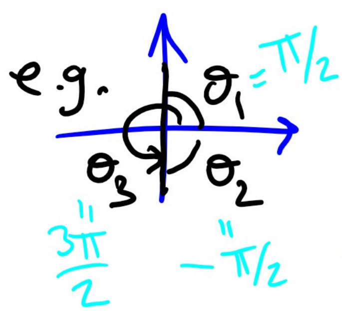
$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \ldots$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \ldots$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \ldots$$



What happens if $x = 0$?

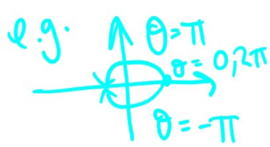


$\tan\theta$ not defined

also $\sec\theta$ not defined

& $\cos\theta =$ ~~$\cot\theta$~~ $= 0$

We'll come back to this later on!!!



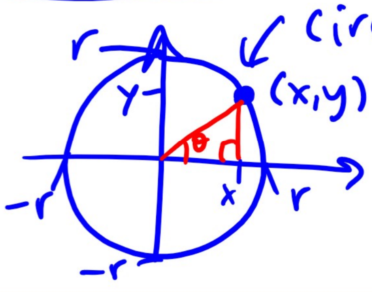
& if $y = 0$?

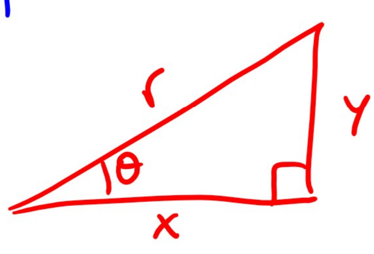
~~$\cot\theta$ &~~ $\csc\theta$ are not defined

undefined if $x \neq 0$, $y = 0$

If $x = 0$ too … we'll come back to this!!!

$$\sin\theta = \tan\theta = 0$$





$$x^2 + y^2 = r^2$$

$$\left.\begin{array}{l} r\sin\theta = y \\ r\cos\theta = x \end{array}\right\} \rightarrow \cancel{r^2}\cos^2\theta + \cancel{r^2}\sin^2\theta = \cancel{r^2}$$

$$\underline{\cos^2\theta + \sin^2\theta = 1}.$$

Trig. Identities

We can also that $\underline{\tan\theta = \dfrac{\sin\theta}{\cos\theta}}$.

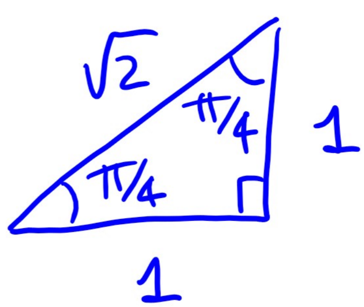
From these we get $\dfrac{\cos^2\theta}{\sin^2\theta} + \dfrac{\sin^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta}$

$$\underline{\cot^2\theta + 1 = \csc^2\theta}$$

& we can also get $\underline{1 + \tan^2\theta = \sec^2\theta}$

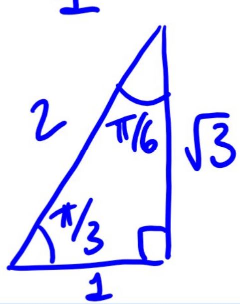
Really useful! Remember this!!

Examples: Special Triangles



$$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}$$

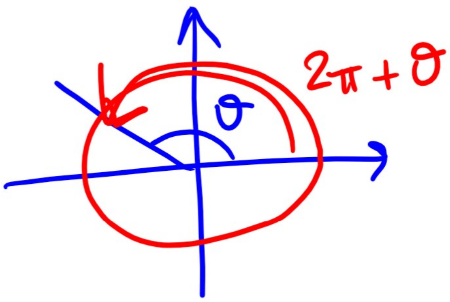
$$\tan\frac{\pi}{4} = 1 \quad \left(= \frac{\sin \pi/4}{\cos \pi/4} \right)$$



$\sin \pi/3 = \sqrt{3}/2$ $\qquad$ $\sin \pi/6 = 1/2$

$\cos \pi/3 = 1/2$ $\qquad$ $\cos \pi/6 = \sqrt{3}/2$

$\tan \pi/3 = \sqrt{3}$ $\qquad$ $\tan \pi/6 = 1/\sqrt{3}$



$$\sin\theta = \sin(2\pi+\theta)$$

$$\cos\theta = \cos(2\pi+\theta)$$

We say sin and cos are 2π-periodic.