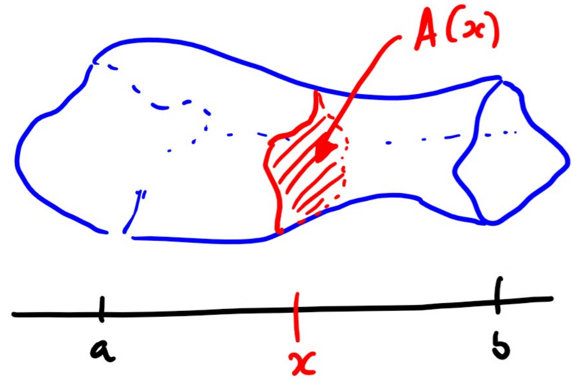
1A03 – CALCULUS I FOR SCIENCE
(SECTION C02)

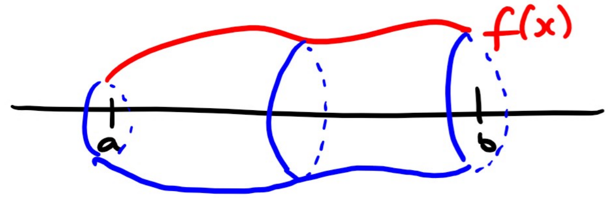
Lecture 29

Last time — VOLUMES

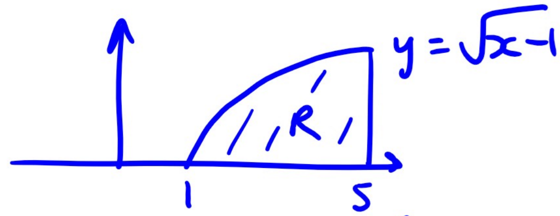
$$\text{Volume of solid} = \int_a^b A(x)\,dx$$

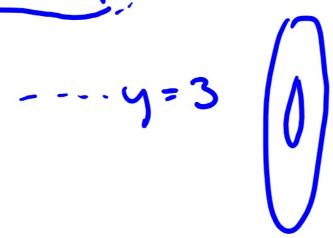


$$\text{Volume of solid of revolution} = \int_a^b \pi (f(x))^2\,dx$$

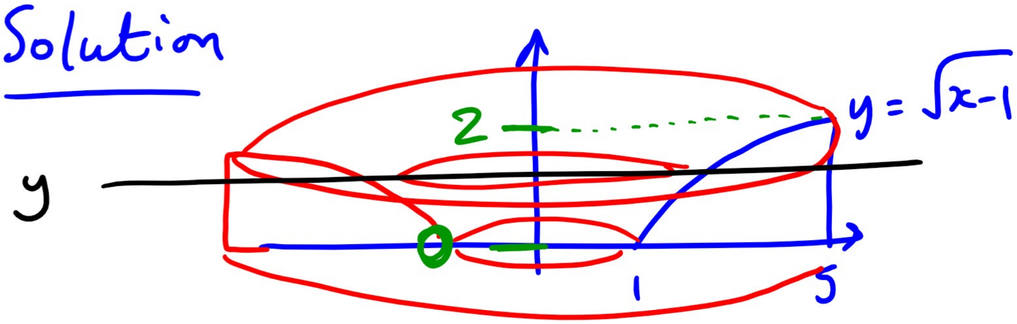


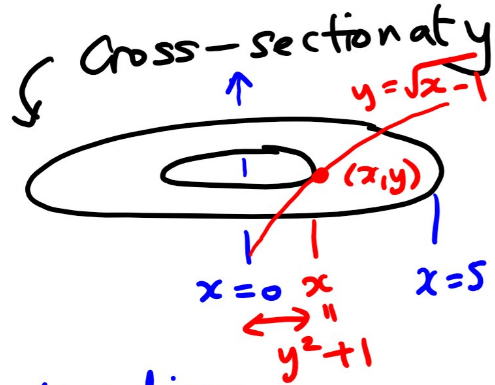
Example Rotate R around y-axis & find the volume of solid generated.





Solution





Area of cross-section $A(y)$ = area of outer disc − area of inner disc

$$= \pi(5)^2 - \pi(y^2+1)^2$$

$$\text{Volume} = \int_0^2 25\pi - \pi(y^2+1)^2\,dy = \pi\int_0^2 25 - y^4 - 2y^2 - 1\,dy$$

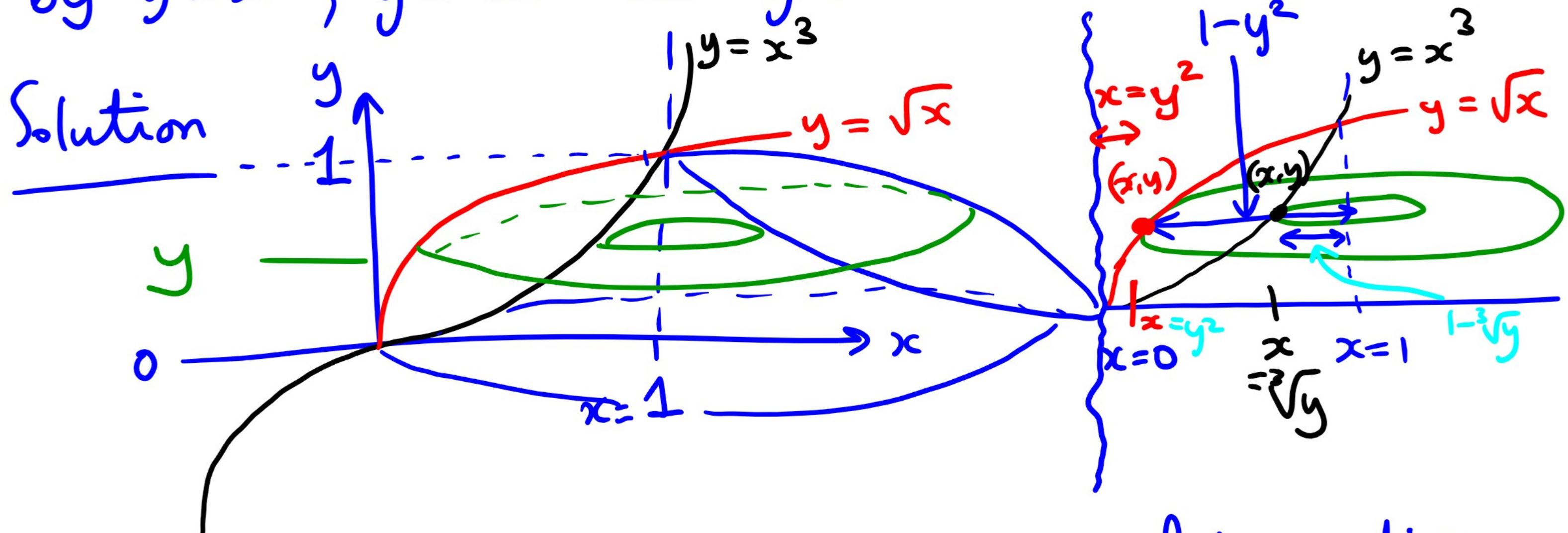
Integrate from $y=0$ to $y=2$

$$= \pi\left[25y - \frac{y^5}{5} - \frac{2y^3}{3} - y\right]_0^2$$

$$= \ldots = \frac{544\pi}{15}.$$

<u>Example</u> Find the volume of the solid generated by rotating the region bounded by $y=x^3$, $y=\sqrt{x}$ on $y \geq 0$ around $x=1$.

<u>Solution</u>



Area $A(y)$ = area of outer disc − area of inner disc

$$= \pi(1-y^2)^2 - \pi(1-\sqrt[3]{y})^2$$

$$\text{Volume} = \pi\int_0^1 (1-y^2)^2 - (1-\sqrt[3]{y})^2\, dy = \ldots = \frac{13\pi}{30}.$$

6.4 WORK

<u>Goal</u>: Find work needed to move an object along a straight line from a to b.

$$\text{Work} = \underbrace{\text{Force}}_{=\text{Mass} \times \text{Acceleration}} \times \text{Distance}$$

Units	Force	Work
Metric	$kg\,m/s^2 = N$ (Newton)	$Nm = J$ (Joule)
Imperial	lb (pound)	ft-lb

Example How much work is done to lift a 15kg aardvark 2m off the ground?

Solution Force needs to counteract gravity

$$g = 9.8 m/s^2$$

$$\text{Work} = \text{Force} \times \text{distance}$$

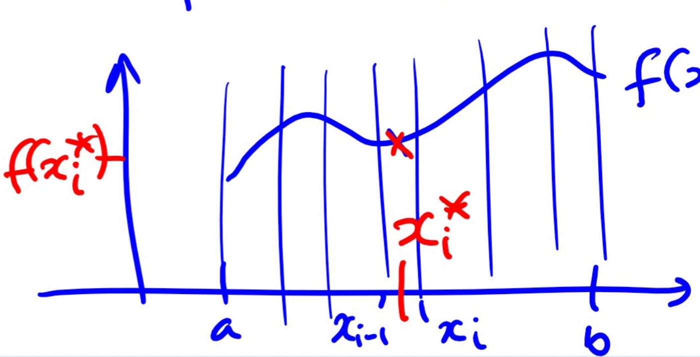
$$= (\underset{\text{mass}}{15} \times \underset{\text{accel.}}{9.8}) \times 2 = \underline{\underline{294\ J}}.$$

But what to do if force is dependent on distance?

e.g. **Hooke's Law** Force required to maintain a spring x units beyond its natural length  is proportional to x.

i.e. Force $= kx$ (k constant).

If we have a formula for force in terms of x i.e. force $= f(x)$, then:



– divide up $[a,b]$ into n subintervals $[x_{i-1}, x_i]$ of equal length

$$\Delta x = \frac{b-a}{n}.$$

- If x_i^* is a sample point in $[x_{i-1}, x_i]$, then $f(x_i^*)$ approximates $f(x)$ on $[x_{i-1}, x_i]$.

- Work done moving object from x_{i-1} to x_i is

$$= \text{Force} \times \text{distance} \approx f(x_i^*)\Delta x$$

- Total work to move object from a to b is

$$\approx \sum_{i=1}^{n} f(x_i^*)\Delta x$$

- Total work $= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x \overset{\text{By definition}}{=} \int_a^b f(x)\,dx.$

<u>Example</u> Suppose that force of 10N needed to stretch a spring 5cm from its natural length. How much work is done in stretching it 15 cm?

<u>Solution</u> By Hooke's Law, force $f(x) = kx$, some k.

To find k: we know $f(0.05) = 10$

i.e. $k(0.05) = 10 \Rightarrow k = \dfrac{10}{0.05} = 200.$

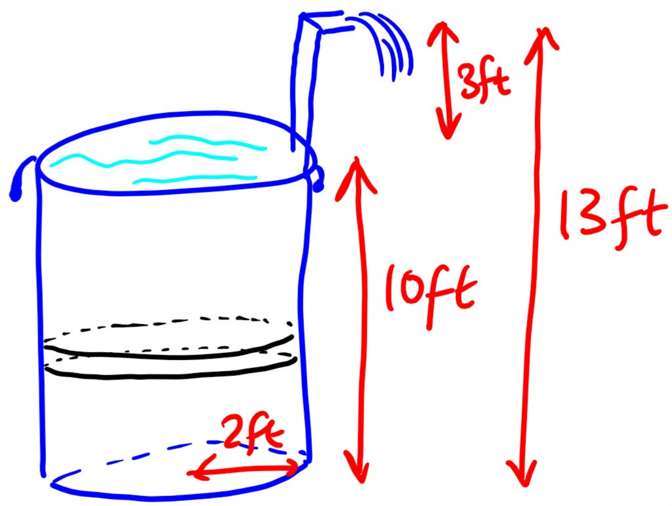
So $f(x) = 200x$.

Total work $= \displaystyle\int_0^{0.15} 200x\,dx = \left[100x^2\right]_0^{0.15} = \underline{\underline{2.25\text{ J}}}.$

But what if there is no nice formula for the force $f(x)$? Then we need to be creative!

Example A cyclindrical tank, height 10ft, radius 2ft, is filled with water. How much work is needed to pump the water up to a spout 13ft high (off the ground)?

Solution



Where is the distance being measured?

Where do you want to put your axes?

Once you've decided what "distance x" means, how much mass is there located at x?