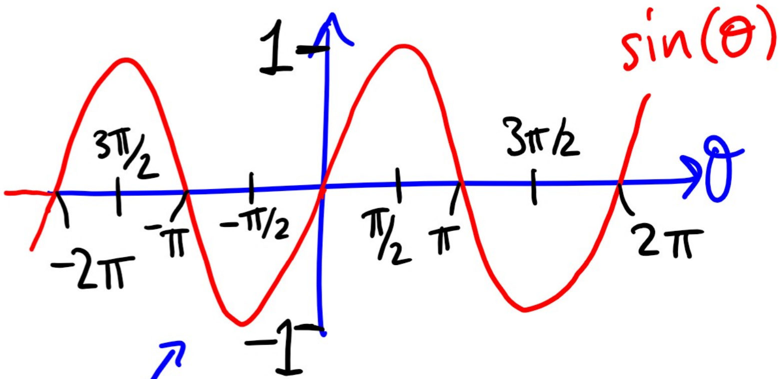
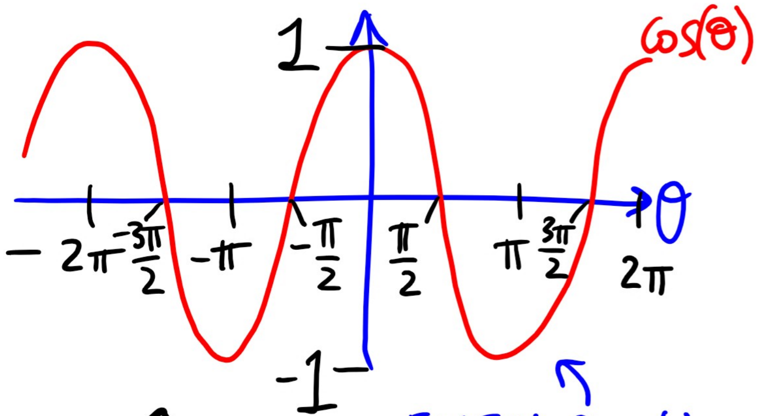
1A03 – CALCULUS I FOR SCIENCE

(SECTION C02) Lecture 3

Last time





ODD function

$\sin(-\theta) = -\sin(\theta)$

2π – periodic

EVEN function

$\cos(-\theta) = \cos(\theta)$.

Addition Formulae

$$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$

$$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$

In particular,

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

Double Angle Formulae

$$(= 2\cos^2\theta - 1)$$

$$(= 1 - 2\sin^2\theta).$$

Remember all your trig. identities. They will be very useful later, especially when it comes to integration!!!

1.5 Inverse Functions & Logarithms

$y = f(x)$

f function

x "input" independent variable

y "output" dependent variable

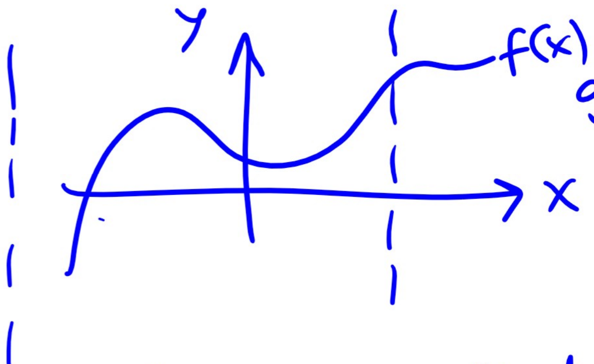
set of all useable x (i.e. you can plug into f)

$=$ domain(f)

$= \text{dom}(f) = D(f)$

set of all outputs ever produced by f = range of f

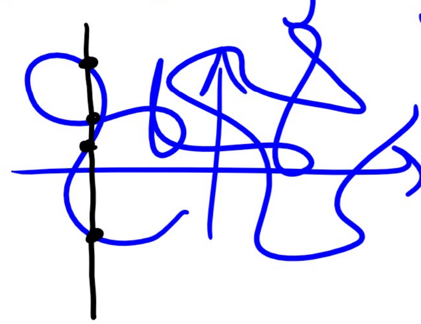
$= \text{ran}(f) = R(f)$.



graph of a function passes vertical line test (VLT)

i.e. every input has exactly one valid output

i.e. any vertical line passes through "graph" at most once.



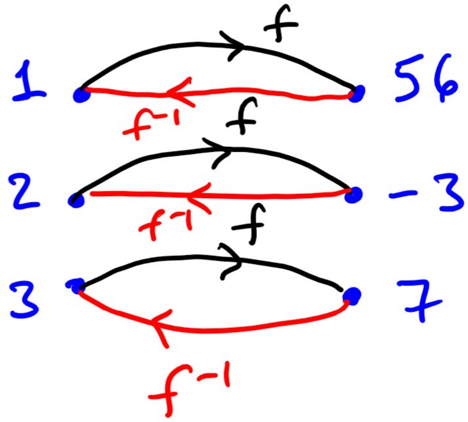
FAIL

not a graph

Inverse Functions

: "undoing" functions

functions "reversed".



$\text{dom}(f) = \{1, 2, 3\}$

$\text{ran}(f) = \{56, -3, 7\}$

The inverse of f, called f^{-1} "f inverse"

reverses f

$\underline{\mathrm{dom}(f^{-1})} = \{56, -3, 7\} = \underline{\mathrm{ran}(f)}$
$\underline{\mathrm{ran}(f^{-1})} = \{1, 2, 3\} = \underline{\mathrm{dom}(f)}$ } general rule

[f^{-1} does NOT mean $\frac{1}{f}$!!!
$f^{-1}(x)$ does NOT mean $\frac{1}{f(x)} = (f(x))^{-1}$]

What is $f^{-1}(f(x))$ for any $x \in \mathrm{dom}(f)$?
$= x$

"belongs to"
"in"

Similarly $f(f^{-1}(x)) = x$ for any $x \in \mathrm{dom}(f^{-1})$.

So $(f^{-1})^{-1} = f$. So the inverse of f^{-1} is f.

How do we find f^{-1} for a given function f?

__Example__ Find $f^{-1}(x)$ when $f(x) = \frac{1}{7+x}$.

__Solution__ __Step 1__ Write $y = \frac{1}{7+x}$.

It's OK to rename the variables first and then solve for x.

__Step 2__ Solve for x: $(7+x)y = 1$

$$\Rightarrow 7y + xy = 1$$

"implies"

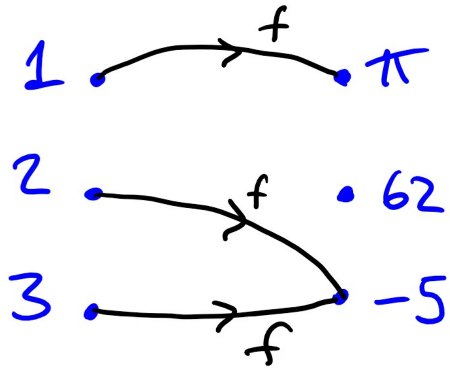
$$\Rightarrow xy = 1 - 7y$$

$$\Rightarrow x = \frac{1-7y}{y}$$

← This would be $f^{-1}(y)$.

__Step 3__ $\underline{f^{-1}(x) = \frac{1-7x}{x}}$. ← Rename ys to xs to get $f^{-1}(x)$.

Example



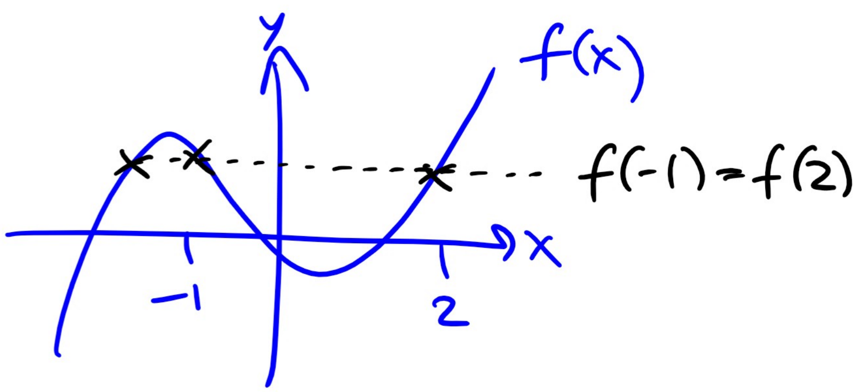
What is $f^{-1}(x)$ for each $x \in \text{ran}(f)$?

$\text{ran}(f) = \{\pi, -5\}$

$\cancel{(\{\pi, -5, -5\})}$ we won't repeat elements

Bad question! No! There is NO f^{-1} here!!

In order to be able to define f^{-1} for f we need that f is called <u>one-to-one</u> (1-1) i.e. no two inputs get sent to the same output.



f is NOT 1-1

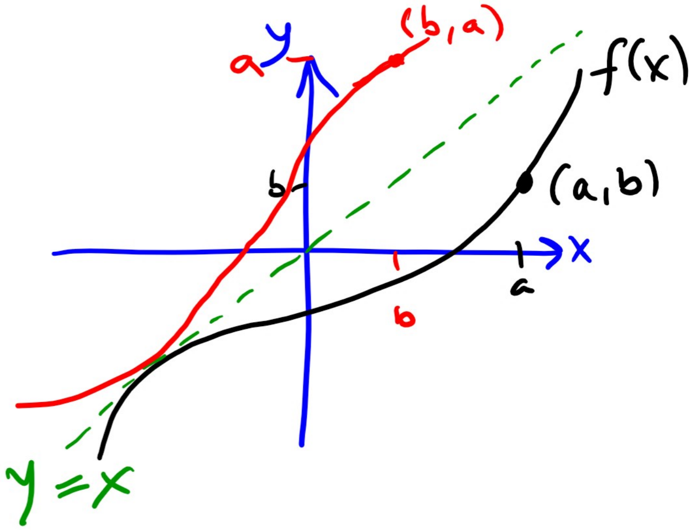
– no inverse

A function is 1-1 exactly when it passes the <u>Horizontal Line Test</u> (HLT): any horizontal line passes through graph(f) at most once.

<u>If</u> f is 1-1, so f^{-1} can be defined, what is the graph of f^{-1}?

Well if $f(a) = b$, then $f^{-1}(b) = a$

i.e. (a,b) is a point on graph(f), then (b,a) is a point on graph(f^{-1})



i.e. graph(f^{-1}) is the reflection of graph(f) in the line $y = x$.

Next time: logs, inverse trig functions.