1A03 – Calculus I for Science

(Section C02) Lecture 37

Last time: Integration Strategy

1. Simplify (multiply out, cancel, rewrite).
2. Obvious Substitution ($u = g(x)$ where $g'(x)$ is a factor) [5.5]
3. Classify
   - (a) Trig. integrals ($\cos^3 x \sin^6 x$, $\tan^2 x \sec^5 x$, $\cos 2x \sin 3x$) [7.2]
   - (b) Rational functions ($\frac{P(x)}{Q(x)}$) [7.4]
   - (c) Radicals
     - $\sqrt{\pm a^2 \pm x^2}$: trig. substitution [7.3]
     - $\sqrt[n]{x}$: try substitution $u = \sqrt[n]{x}$
   - (d) Integration by Parts: $u\,dv$ — u gets simpler when differentiated; dv can be integrated [7.1]

Integration Examples

1. Inspiration: Prob. Sampler #3 Q8.

Find
$$\int_0^{2\pi} 3^x \cos(x)\,dx = \cancel{\left[3^x \sin(x)\right]_0^{2\pi}} - \int_0^{2\pi} (\ln 3) 3^x \sin(x)\,dx$$

Integration by Parts:

$u = 3^x = e^{\ln 3^x} = e^{x\ln 3}$ $\quad dv = \cos(x)\,dx$

$\frac{du}{dx} = (\ln 3)3^x$ $\quad v = \sin(x)$

$t = 3^x$ $\quad \frac{ds}{dx} = \sin(x)$

$\frac{dt}{dx} = (\ln 3)3^x$ $\quad s = -\cos(x)$

$$= -\ln 3\left(\left[-3^x \cos(x)\right]_0^{2\pi} + \int_0^{2\pi} (\ln 3)\,3^x \cos(x)\,dx\right)$$

$$= -\ln 3\left(-3^{2\pi} - (-3^0) + \ln 3 \int_0^{2\pi} 3^x \cos(x)\,dx\right)$$

If we let $A = \int_0^{2\pi} 3^x \cos(x)\,dx$, then we just showed:

$$A = -\ln 3\,(1 - 3^{2\pi}) - (\ln 3)^2 A$$

Solve for A: $\quad (1 + (\ln 3)^2)A = \ln 3\,(3^{2\pi} - 1)$

$$A = \frac{\ln 3\,(3^{2\pi} - 1)}{1 + (\ln 3)^2}$$

Take-home message: you can use the cyclical nature of the (anti) derivatives of $\sin(x)$ & $\cos(x)$ together with integration by parts to obtain an equation to solve for the integral.

This is all OK! I got confused in class as my notes were wrong

---

② Inspiration PS#3 Q16.

$$\int \frac{dx}{x\sqrt{9+x^2}} = \int \frac{\cancel{3}\sec^{\cancel{2}}\theta\,d\theta}{3\tan\theta\,\cancel{3\sec\theta}} = \frac{1}{3}\int \frac{\sec\theta}{\tan\theta}\,d\theta$$

$x = 3\tan\theta$

$dx = 3\sec^2\theta\,d\theta$

$$= \frac{1}{3}\int \frac{1}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{\sin\theta}\,d\theta$$

$$= \frac{1}{3}\int \csc\theta\,d\theta.$$

Also

$$\int \csc^2\theta\,d\theta = -\cot\theta + C$$

$$= \frac{1}{3}\ln|\csc\theta - \cot\theta| + C$$

Take-home message: Know your basic integrals! (p. 503)

$$= \ldots$$

---

③ Inspiration PS#3 Q21

Find $\int \underbrace{\ln(x^2+5)}\,dx$

Could try trig. sub. but could gets ugly

$x = \sqrt{5}\tan\theta \rightarrow \int \ln(5\sec^2\theta)\sqrt{5}\sec^2\theta\,d\theta$

What about I.P.?

Take-home: sometimes useful even with one function

$u = \ln(x^2+5) \qquad dv = dx$

$du = \dfrac{2x}{x^2+5} \qquad v = x$

$\to x\ln(x^2+5) - \underbrace{\int \frac{2x^2}{x^2+5}\,dx}$

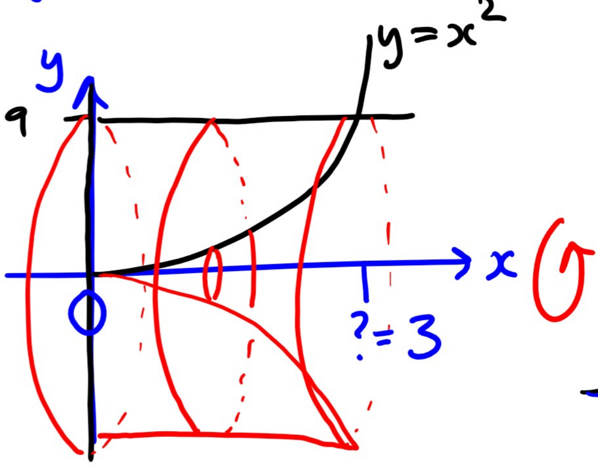
$\to$ Rational Function

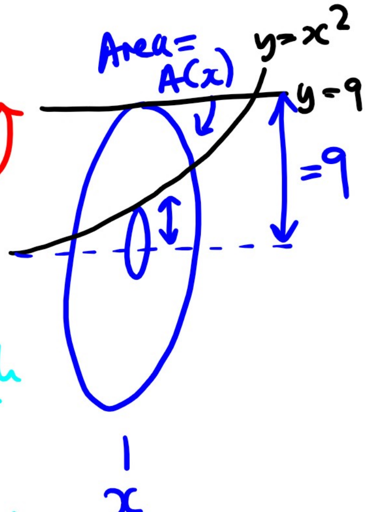
Use the algorithm.

Volumes of Solids of Revolution

Inspiration : PS#3 Q5

Find the volume of solid obtained by rotating the region bounded by the y-axis, $y=x^2$ and $y=9$ around the x-axis.





$$\text{Volume} = \int_0^3 A(x)\,dx$$

$A(x)$ = area of outer disc − area of inner disc

$= \pi \cdot 9^2 - \pi(x^2)^2$

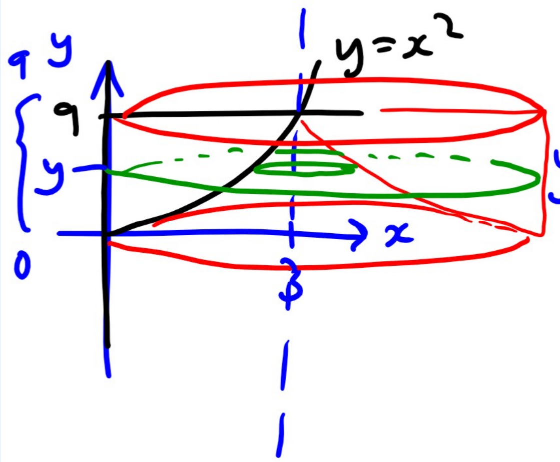
$= 81\pi - \pi x^4$

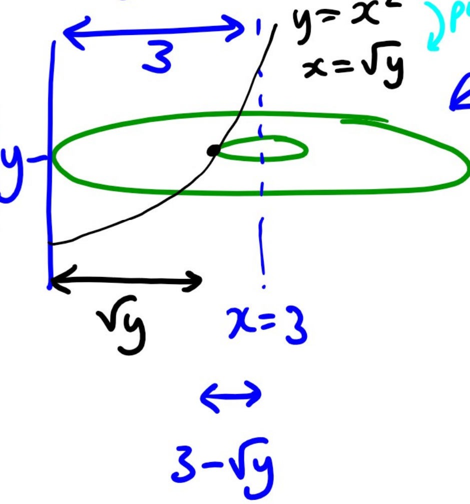
Take-home message in both parts: use the curves that describe the rotated region to work out radii of inner & outer circles (in terms of the correct variable!)

So volume $= \displaystyle\int_0^3 81\pi - \pi x^4\,dx = \left[81\pi x - \frac{\pi x^5}{5}\right]_0^3$

$= 243\pi - 243/5\,\pi = 194.4\pi$.

Now rotate our region around $x = 3$





put the equation in terms of y

Area = area of outer disc − area of inner disc

$= \pi . 3^2 - \pi(3-\sqrt{y})^2$

$$\text{Volume} = \int_0^9 \pi 3^2 - \pi(3-\sqrt{y})^2 \, dy .$$

$$= \int_0^9 9\pi - \pi(9 - 6\sqrt{y} + y) \, dy$$

$$= \pi \int_0^9 6\sqrt{y} - y \, dy .$$

FINAL THOUGHTS:

① COURSE EVALUATIONS: I know you're all very busy but, if you possibly can, please take a few minutes break before the end of Thursday to review this course — help us to improve it for the future (as well as tell us what we're doing right!). Link on course website & my 1A03 website.

② You've got this! Do your best - it's all anyone can ask. GOOD LUCK!!