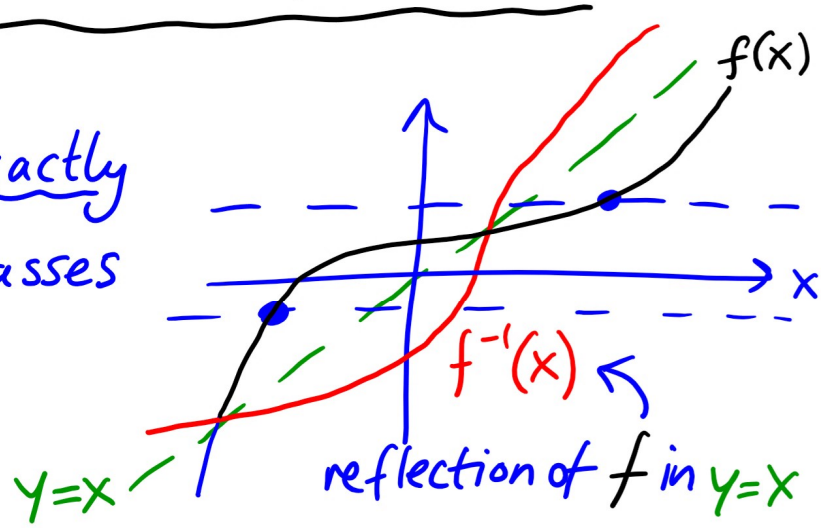
1A03 – Calculus I for Science (Section C02)

Lecture 4

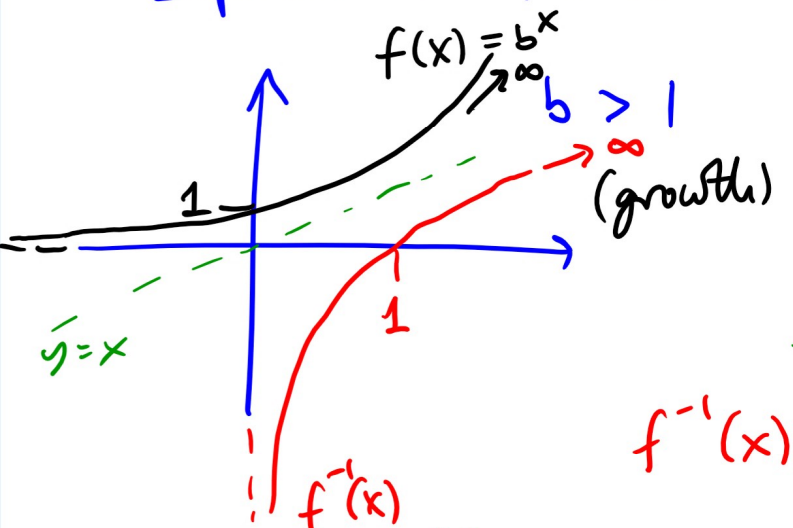
Last time Inverse and 1-1 Functions

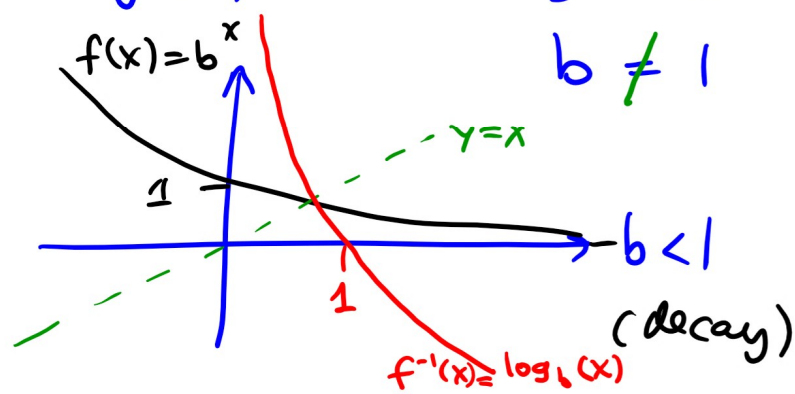
f has an inverse f^{-1} exactly when it is 1-1 i.e. passes the Horizontal Line Test:



Logarithmic Functions

Exponential functions: $f(x) = b^x$ $b > 0$, $b \neq 1$





$f^{-1}(x) = \log_b x$ – exponent to which b must be raised to get x

$b^{\log_b x} = x = \log_b(b^x)$

Crucial example: Natural logarithm $b = e \approx 2.718\ldots$ (Euler's constant)

$\log_e(x)$ written $\ln(x)$; $x = e^{\ln(x)} = \ln(e^x)$

Warning: log ≠ log !!

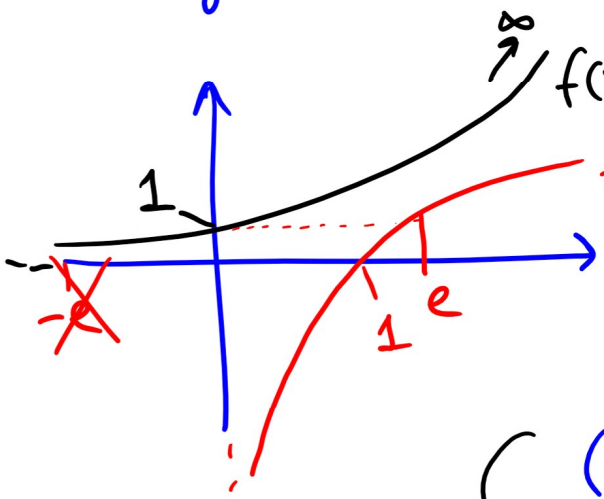
log = $\log_{10}$ for engineering software

log = ln for math. software

Change of base formula

For any $b > 0$, $b \neq 1$ and any $x \in \mathbb{R}$ (real #s)

$$\log_b(x) = \frac{\ln(x)}{\ln(b)} = \left(\frac{1}{\ln(b)}\right)\ln(x)$$



Log Rules

(Also work for $\log_b$)

(1) $\ln(xy) = \ln(x) + \ln(y)$

(2) $\ln(x^r) = r\ln(x)$

(3) $\ln(x/y) = \ln(x) - \ln(y)$

Example Find x when $\ln(10x) = 6 + \ln(x^2)$.

Solution

$\Rightarrow$ ("implies")

$\ln 10 + \ln x = 6 + 2\ln x$

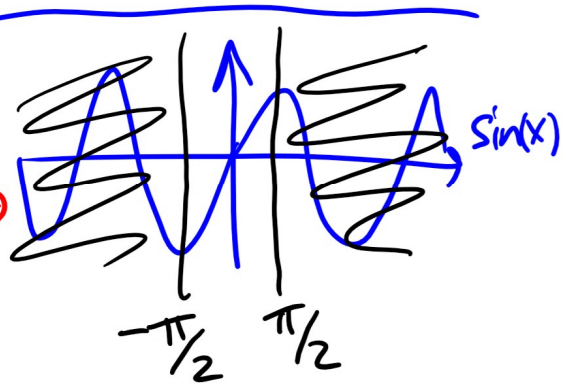
$\ln 10 - 6 = \ln x$

$$e^{\ln 10 - 6} = e^{\ln x} = x$$

$$e^{\ln 10} \cdot e^{-6} = x$$

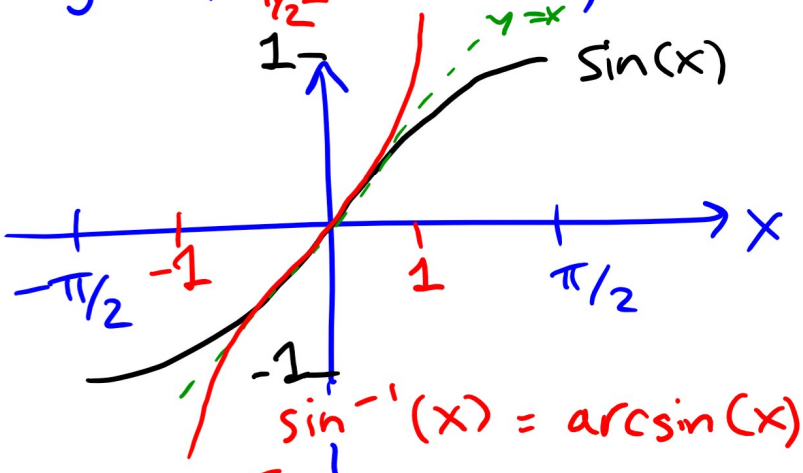
$$10 \cdot e^{-6} = x$$

Inverse Trig. Functions

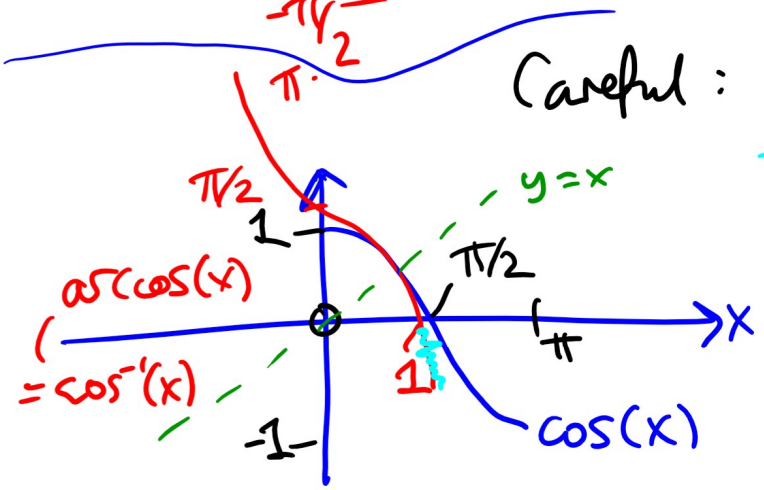


If graph (f) fails HLT
then restrict domain to something where it passes HLT

e.g. for $\sin(x)$, restrict to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ← include endpoints



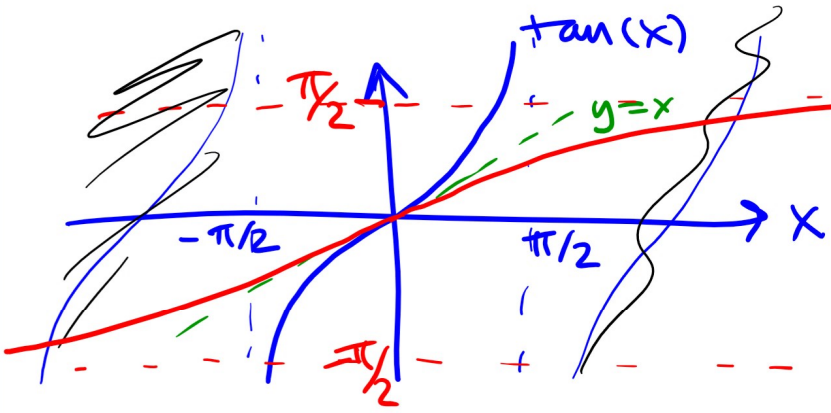
$\sin^{-1}(x) = \arcsin(x)$ (NOT $(\sin(x))^{-1} = \csc(x)$)



Careful: $\arcsin\left(\sin\left(\frac{3\pi}{2}\right)\right) = \arcsin(-1)$

The output of arcsin must be in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $= -\pi/2$

Again: care required
cos restricted to $[0, \pi]$
(so output of arccos must be in $[0, \pi]$)



$\arctan(x) = \tan^{-1}(x)$

Again careful:
tan restricted to $(-\frac{\pi}{2}, \frac{\pi}{2})$

(So, output of arctan must be in $(-\frac{\pi}{2}, \frac{\pi}{2})$.) not including endpoints

Example Simplify $\sin(\underbrace{\tan^{-1}(x)}_{y \text{ or } \theta})$.

Solution

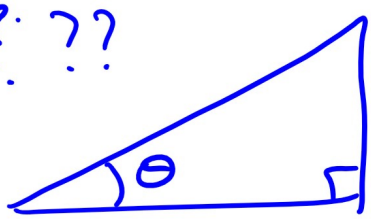
Now question is: simplify $\sin\theta$ (in terms of x)
i.e. find an expression for $\sin\theta$ in terms of x

$\theta = \tan^{-1}(x)$

$\tan\theta = x$

$= \frac{\text{opp.}}{\text{adj.}}$??

Draw triangle! $\frac{\text{opp.}}{\text{hyp.}}$??



Answer to be revealed next time!