1A03 – Calculus I for Science
(Section C02)

Lecture 5

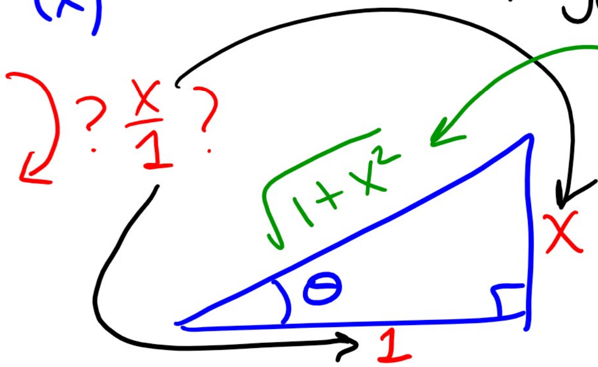
Last Time: **Example** Simplify $\sin(\underbrace{\tan^{-1}(x)}_{=\theta})$.

Start by giving your inverse function a variable name

Solution We want an expression for $\underset{\text{opp./hyp.}}{\sin\theta}$ in terms of x.

$\theta = \tan^{-1}(x)$

$\tan\theta = x = \dfrac{\text{opp.}}{\text{adj.}}$? $\dfrac{x}{1}$?



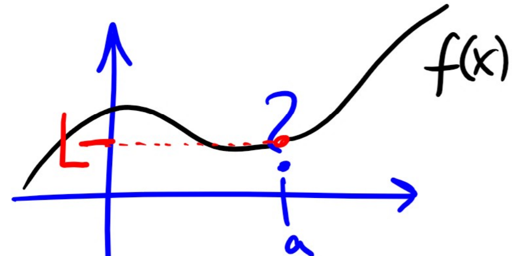
Use Pythagoras!

So $\sin\theta = \dfrac{x}{\sqrt{1+x^2}}$.

i.e. $\sin(\tan^{-1}(x)) = \dfrac{x}{\sqrt{1+x^2}}$.

[§2.5] Limits & Continuity

Limit of $f(x)$ "at a"



We write:

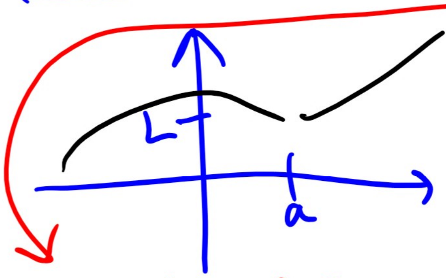
$$\lim_{x\to a} f(x) = L \quad \leftarrow \text{a \#}$$

We know what f does near a (not necessarily at a)

if: we can make the value of $f(x)$ as close to L as we like by taking x to be as close to a as we like

One-sided limits

$$\lim_{x \to a^-} f(x) = L$$

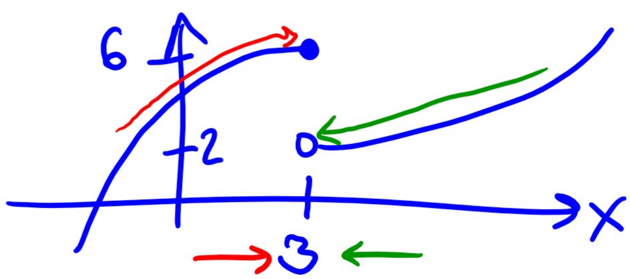


or $$\lim_{x \to a^+} f(x) = L$$

look only at $x < a$

look only at $x > a$

Example



$$\lim_{x \to 3^-} f(x) = 6$$

$$\lim_{x \to 3^+} f(x) = 2$$

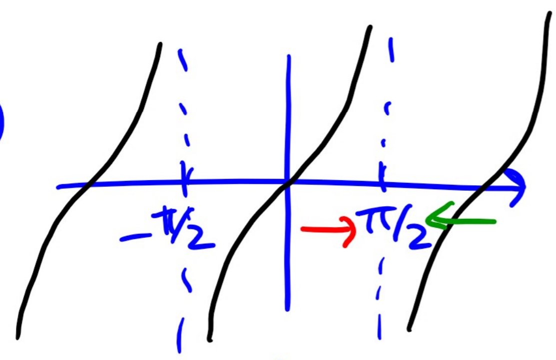
(even though $f(3) = 6$)

$\lim_{x \to 3} f(x)$ DNE (does not exist)

(as $\lim_{x \to 3^-} f(x) \neq \lim_{x \to 3^+} f(x)$).

Infinite Limits

e.g. $\tan(x)$



$$\lim_{x \to \frac{\pi}{2}^-} \tan(x) = \infty$$

?!

We can make $\tan(x)$ as big as we like by taking x as close to $\frac{\pi}{2}$ (on the left / negative side) as we like.

$$\lim_{x \to \frac{\pi}{2}^+} \tan(x) = -\infty$$

(also DNE but we can say more here that just huh?)

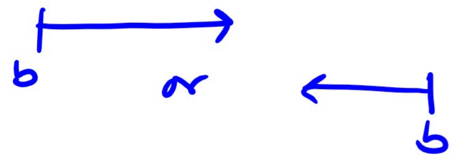
→ we can say in what way the limit DNE

Vertical Asymptotes: $\lim_{x \to a^{\circ}} f(x)$ "=" $\pm\infty$

↖ could put $\pm$ here for one-sided

Limits at Infinity

$f(x)$ defined on some ray (b, ∞) or $(-\infty, b)$



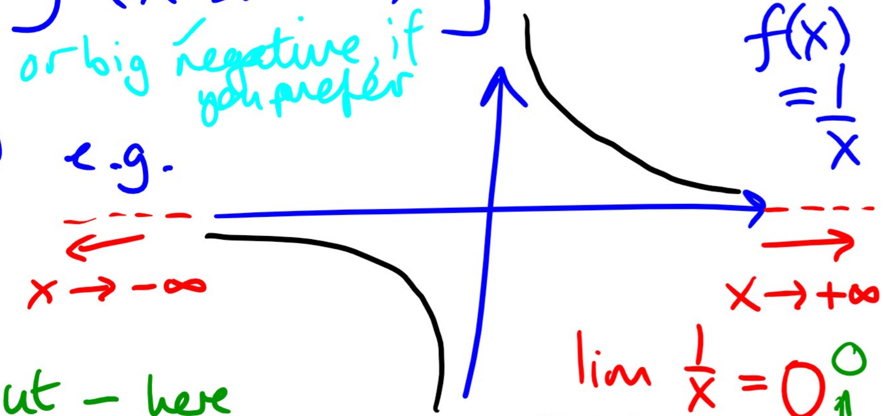
$\lim\limits_{x \to \infty} f(x) = L$ ← #

if we can get $f(x)$ as close to L as we like by taking x as big as we like

[$\lim\limits_{x \to -\infty} f(x) = L$ similarly (x ~~small~~ take small) or big negative, if you prefer]

Horizontal Asymptotes

e.g.



$\lim\limits_{x \to -\infty} \frac{1}{x} = 0^{\ominus}$ ← Could put − here to indicate approaching from below

$\lim\limits_{x \to +\infty} \frac{1}{x} = 0^{\oplus}$ ← Could put +

Limit Laws

If $\lim\limits_{x \to a} f(x)$ & $\lim\limits_{x \to a} g(x)$ exist

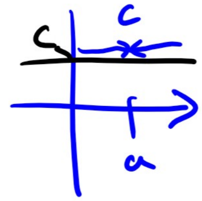
must be same a

→ a could be a # or $\pm\infty$

(a) $\lim\limits_{x \to a} (f(x) \pm g(x)) = \lim\limits_{x \to a} f(x) \pm \lim\limits_{x \to a} g(x)$

(b) $\lim\limits_{x \to a} (f(x) g(x)) = \left(\lim\limits_{x \to a} f(x)\right)\left(\lim\limits_{x \to a} g(x)\right)$

(c) $\lim\limits_{x \to a} (c f(x)) = c \left(\lim\limits_{x \to a} f(x)\right)$

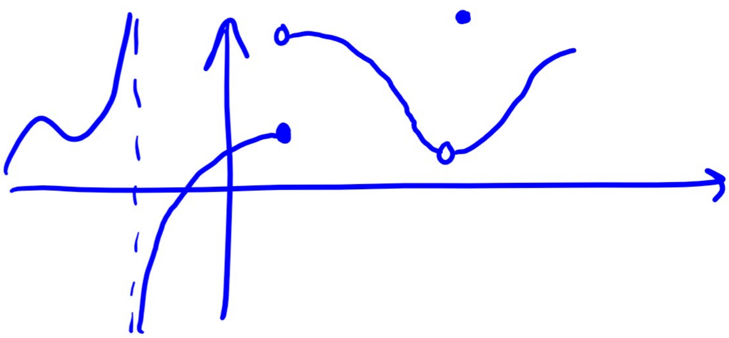


(d) $\lim_{x \to a} \left(f(x) / g(x) \right) = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ ← **IF** $\lim_{x \to a} g(x) \neq 0$

Continuity A function $f(x)$ is continuous at a
↑
a #

if $\lim_{x \to a} f(x) = f(a)$ ← So notice this says

- f defined at a $(a \in dom(f))$
- $\lim_{x \to a} f(x)$ exists (so is a #)
- $\lim_{x \to a} f(x)$ actually equals $f(a)$



← Examples of discontinuities.