1ZA3 (SECTION C01) – ENGINEERING MATHEMATICS I

Lecture 16

Last time

MEAN VALUE THEOREM

If $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b), then there is c in $[a,b]$ with $f'(c) = \dfrac{f(b)-f(a)}{b-a}$ ← average rate of change on $[a,b]$.

INCREASING/DECREASING TEST

If $f(x)$ is differentiable and

$f'(x) \begin{matrix} > \\ < \end{matrix} 0$ on an interval, then $f(x)$ is $\begin{matrix}\text{increasing} \\ \text{decreasing}\end{matrix}$ over that interval.

So $f(x)$ can only switch between increasing & decreasing when $f'(x) = 0$.

FIRST DERIVATIVE TEST

If $f(x)$ is continuous and c is a critical # of $f(x)$ then:

We have a local minimum at $x=c$ if $f'(x)$ changes from negative to positive at $x=c$

&

We have a local maximum at $x=c$ if $f'(x)$ changes from positive to negative at $x=c$.

If $f(x)$ is differentiable, notice "$f'(x)$ changes from $-$ve to $+$ve" means $f'(x)$ is increasing (through value 0) (Similarly for other direction.)

SECOND DERIVATIVE TEST

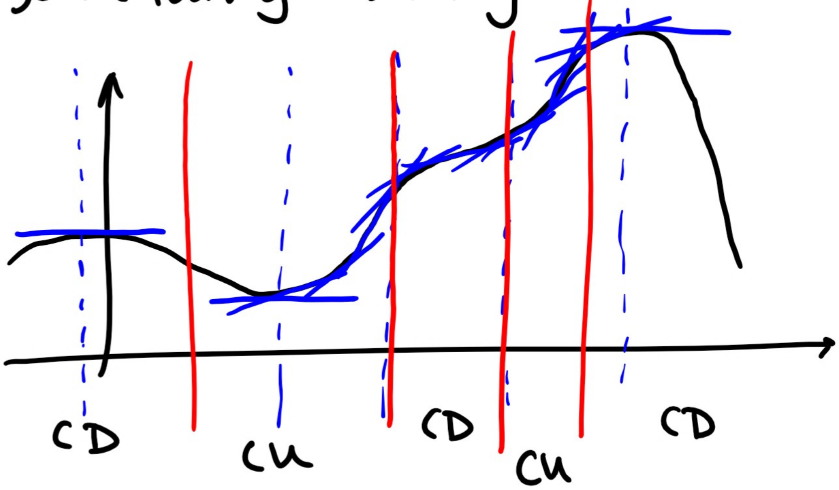
If $f''(x)$ is continuous near $x=c$, then

(a) if $f'(c)=0$ and $f''(c)>0$ then $f(x)$ has a local minimum at $x=c$

(b) If $f'(c)=0$ and $f''(c)<0$ then $f(x)$ has a local maximum at $x=c$

Notice This test does not help us if $f''(c)=0$ or $f''(x)$ not defined at $x=c$.

Extend this idea: second derivative also can tell us something away from critical #s.



Definition

$f''(x)>0$ (f' increasing) we say graph of $f(x)$ is Concave Upward (CU)

and $f''(x)<0$ (f' decreasing) we say graph of $f(x)$ is Concave Downward (CD)

CONCAVITY TEST

If $f''(x)>0$ on an interval, then the graph is CU on that interval

If $f''(x)<0$ on an interval, then the graph is CD on that interval

Definition A point of inflection $(x, f(x))$ is a point

at which the concavity changes (CD to CU, or CU to CD)
(So $f''(x) = 0$ here.) if f'' defined here, as I said in class.

<u>Example</u> Where is the graph of $f(x) = x^5 - 10x^3 + 7$ CU, CD & where are the points of inflection.

<u>Solution</u> Want to know is where $f''(x) \gtrless 0$ so solve

$$f''(x) = 0$$

(or find where f'' is undefined, but $f(x)$ is a polynomial, so f'' defined everywhere)

First: $f'(x) = 5x^4 - 30x^2$

& so $f''(x) = 20x^3 - 60x = 20x(x^2 - 3)$.

So $f''(x) = 0$ when $x = 0$ or $\pm\sqrt{3}$

	$f''(x) \gtrless 0$	CU / CD	$20x$	x^2-3
$(-\infty, -\sqrt{3})$	$-$	~~CU~~ CD	$-$	$+$
$(-\sqrt{3}, 0)$	$+$	~~CD~~ CU	$-$	$-$
$(0, \sqrt{3})$	$-$	~~CU~~ CD	$+$	$-$
$(\sqrt{3}, \infty)$	$+$	~~CD~~ CU	$+$	$+$

So all points $0, \pm\sqrt{3}$ are points of inflection

$$f(x) = \frac{\sin x}{e^x - 1} = \frac{0}{0} \;!!$$

How to determine $f(0)$??

4.4 L'Hospital's Rule

In situations where we have "indeterminate forms" of various types: $\frac{0}{0}$ or $\frac{\infty}{\infty}$ we get stuck trying to apply limit laws:

We want $\lim_{x \to a} \frac{f(x)}{g(x)}$ but $\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$

or $\lim_{x \to a} f(x) = \pm\infty$ AND

$\lim_{x \to a} g(x) = \pm\infty$

then we have an

indeterminate form of type $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

(Warning : what we do will not apply to $\frac{0}{\#} = 0$

or $\frac{\#}{0} =$ undefined $\pm\infty$)