1ZA3 (SECTION C01) – ENGINEERING MATHEMATICS I

Lecture 19

Last time Curve Sketching

(A) Domain of $f(x)$

(B) Intercepts (where is $f(x)=0$? what is $f(0)$)

(C) Symmetry (even, odd, periodic)

(D) Asymptotes (infinite discontinuities; $\lim_{x\to\pm\infty} f(x)$)

(E) Increasing / Decreasing

(F) Local Max. / Min.

(where is $f'(x) =, <, > 0$? where is $f'(x)$ undefined?)

(G) Concavity (where is $f''(x) =, <, > 0$?)

Example Sketch $f(x) = 2xe^{1/x}$.

Solution (Sketch) (A) Not defined at $x=0$.

(B) No y-intercept because $f(0)$ not defined

$f(x)=0$? $\underbrace{e^{1/x}}_{>0}$ but $f(x)\neq 0$ as f not defined at $x=0$

Any exponential function is >0.

(C) No symmetry (check $f(-x)$). Also no indication of periodicity.

(D) VA: check discontinuities; here $x=0$ is the only discontinuity

Have to check $\lim_{x\to 0^+} f(x)$ and $\lim_{x\to 0^-} f(x)$

$$\lim_{x\to 0^+} \underbrace{2x}_{0}\,\underbrace{e^{1/x}}_{\infty} = \underset{\text{L'H.}}{\dots} = \infty$$

Exercise

Indeterminate product.

$$\lim_{x\to 0^-} \underbrace{2x}_{0}\,\underbrace{e^{1/x}}_{\frac{1}{\infty}\text{ "="} 0} = 0$$

So VA only on one side (+ve side)

HA: $\lim\limits_{x\to\pm\infty} \underbrace{2x}_{\pm\infty}\underbrace{e^{1/x}}_{1} = \pm\infty$ No HA at $\pm\infty$

See below curve sketch for what's really going on as x goes to $+\infty$.

(E) Don't forget, 0 will be a critical # ($f(x)$ not defined at $x=0$)

We also check $f'(x)=0$ ... get $x=1$ (check)

So need to see what happens with $f'(x)$ on

$(-\infty, 0)$, $(0,1)$, $(1,\infty)$

$f'(x)$: >0 <0 >0 (check)

f not defined (so cannot be a local max./min.)

local. min.

(F)

(G) Check $f''(x)=0$ - no solutions

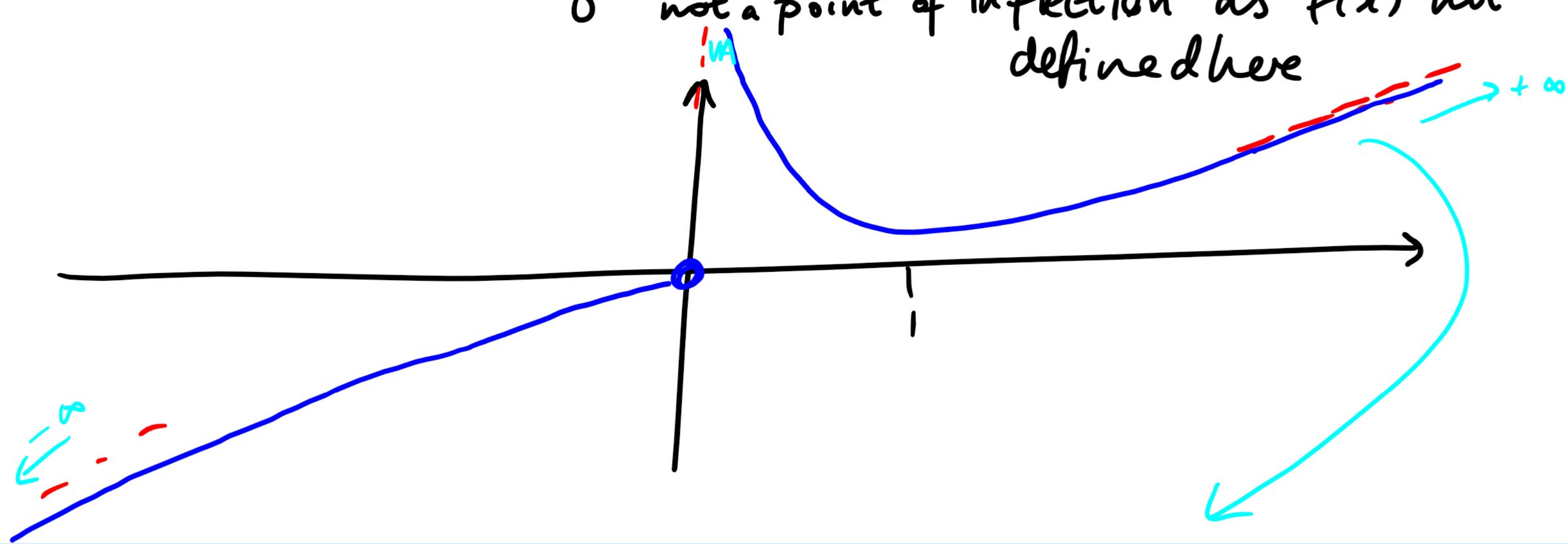
But $f''(x)$ not defined at $x=0$ ($f(x)$ not def.!)

Check $(-\infty, 0)$, $(0,\infty)$

$f''(x)$ <0, >0 (Check)

CD CU

0 not a point of inflection as $f(x)$ not defined here



$f(x) = 2xe^{1/x}$ has a slant asymptote at $x = +\infty$

$$\lim_{x\to\infty}\left(2xe^{1/x} - (2x+2)\right) = 0 \qquad \text{(Check.)}$$

In general, $g(x)$ has a slant asymptote at $\pm\infty$ if there is a line $y = mx + c$ with $\lim_{x\to\pm\infty}\left(g(x) - (mx+c)\right) = 0$.

How to find? $m = \lim_{x\to\infty} g'(x)$ and then use this equation to solve for c.

4.7 Optimization

- Practical application of find absolute maximum / minimum
- Want to minimise/maximise $f(x)$ which maybe represents costs / profits / travel time
  (min.) (max.) (min.)
- Need to translate word problems into equations

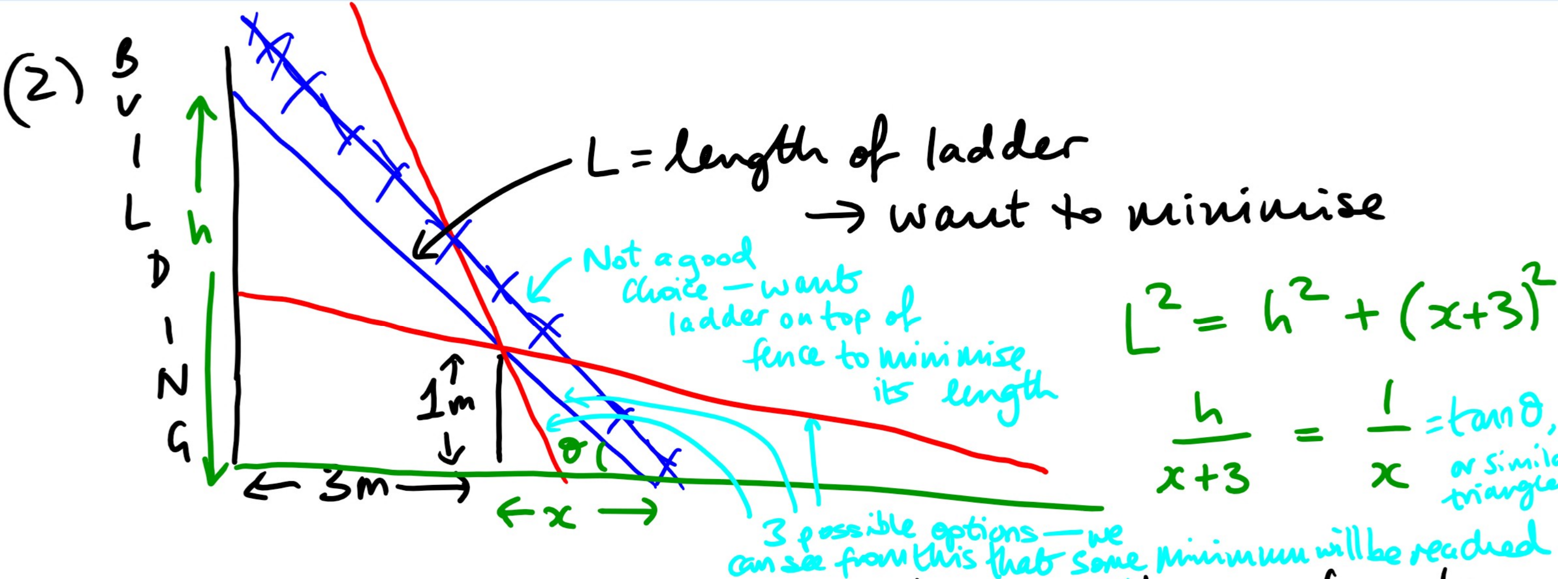
Example A 1m high fence runs alongside a building at a distance of 3m away. What is the length of the shortest ladder to clear the fence & lean against the building?

Solution (1) Understand unknowns, given quantities, constraints (restrictions, conditions)

length of ladder; height of fence; distance of fence from building

ladder must clear fence, rest against building

(2)



$$L^2 = h^2 + (x+3)^2$$

$$\frac{h}{x+3} = \frac{1}{x} = \tan\theta,$$ or similar triangles

Goal: get enough equations to see L as function of one variable:

$$h = \frac{x+3}{x} \quad \text{so} \quad L^2 = \left(\frac{x+3}{x}\right)^2 + (x+3)^2$$

$$= (x+3)^2\left(\frac{1}{x^2} + 1\right)$$

Find abs. min. of L.

It's enough to find abs. min. of L^2 (& then take $\sqrt{}$).