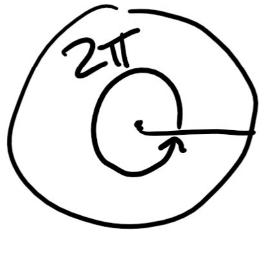
1ZA3 (SECTION C01) – ENGINEERING MATHEMATICS I

Lecture 2

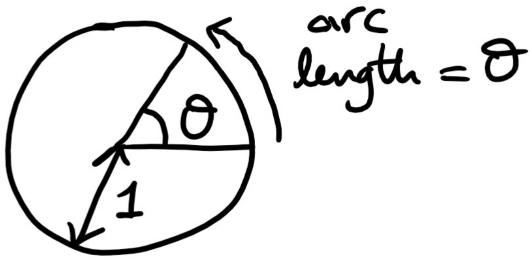
TRIGONOMETRY

From now on: radians not degrees

There are 2π of them in a circle



In fact if you take a unit circle



(so if $\theta = 1$, arc length $= 1$)

So 2π rad $= 360^\circ$

To go from rad $\rightarrow$ deg, divide by 2π & multiply by 360

i.e. $\times \frac{360}{2\pi} = \times \frac{180}{\pi}$

To go from deg $\rightarrow$ rad: $\times \pi/180$.

Example Find 45° in radians.

Solution $45 \times \pi/180 = \pi/4$.

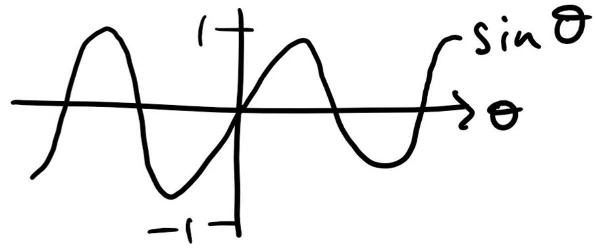
Example Find $\pi/3$ rad in degrees.

Solution

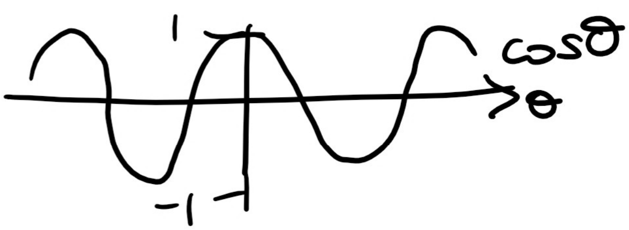
$$\frac{\pi}{3} \times \frac{180}{\pi} = 60^\circ.$$

Trigonometric Functions sin, cos, tan

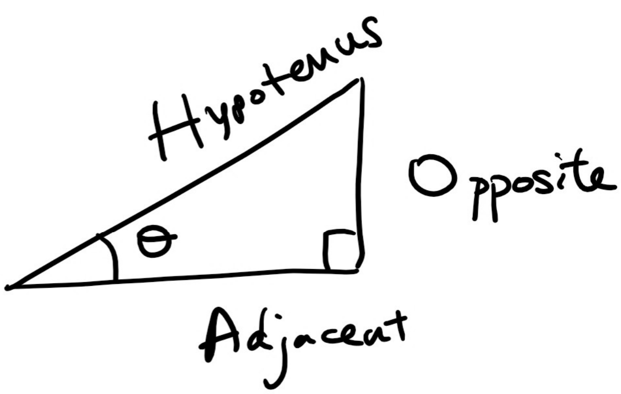
csc, sec, cot



oscillations



How to define them?



SOH CAH TOA

Students of Hamilton

Care about ... ? Send me your suggestions!

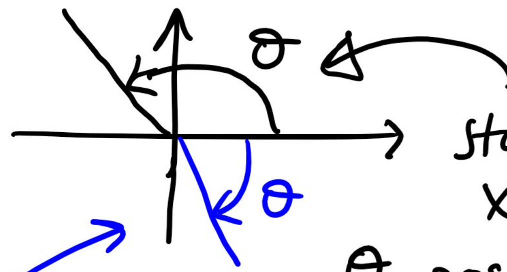
$$\sin\theta = O/H \qquad \tan\theta = O/A \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{H}{O}$$

$$\cos\theta = A/H \qquad \sec\theta = \frac{1}{\cos\theta} = H/A$$

$$\cot\theta = 1/\tan\theta = A/O$$

IF angle θ is acute

If θ is (not) acute, we look at the standard position of θ:

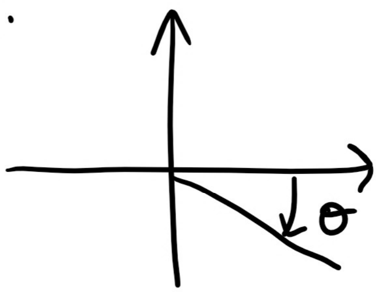


start at positive x-axis & if θ positive, rotate through angle θ counter-clockwise

if θ negative, rotate through angle $|\theta|$ clockwise

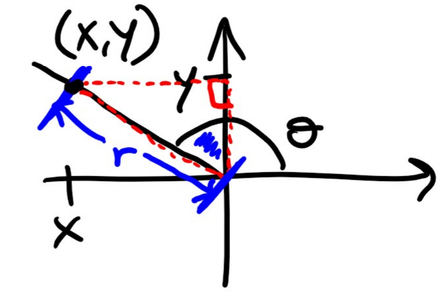
i.e. "$-\theta$" the "positive value" of θ

e.g.



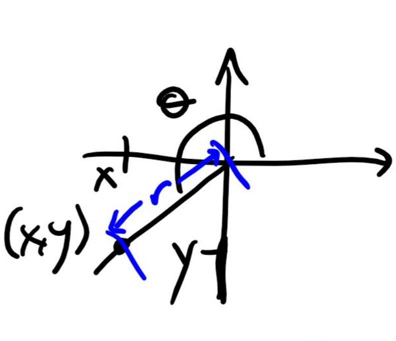
$\theta = -\pi/4$

Then to define trig. functions:



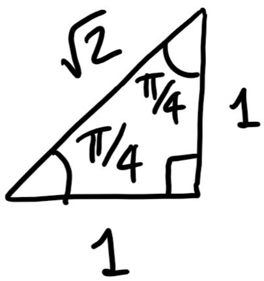
$$\sin\theta = \frac{y}{r} \qquad \tan\theta = y/x$$

$$\cos\theta = \frac{x}{r}$$



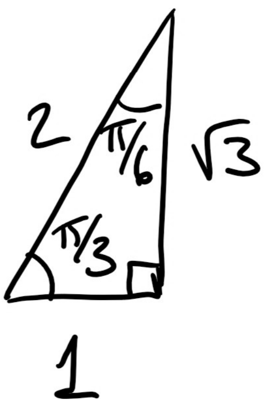
These formulae extend the idea of using SOHCAHTOA & right triangles to any situation i.e. any angles. First draw the line θ given by θ, determine r, x & y, and use these formulae. But if you prefer to pick a right triangle & either use the periodicity of trig. functions or the $\frac{S \mid A}{T \mid C}$ rule to get the correct sign, then that is OK!!!

Examples — Special Triangles



$$\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}.$$

$$\tan\frac{\pi}{4} = 1.$$

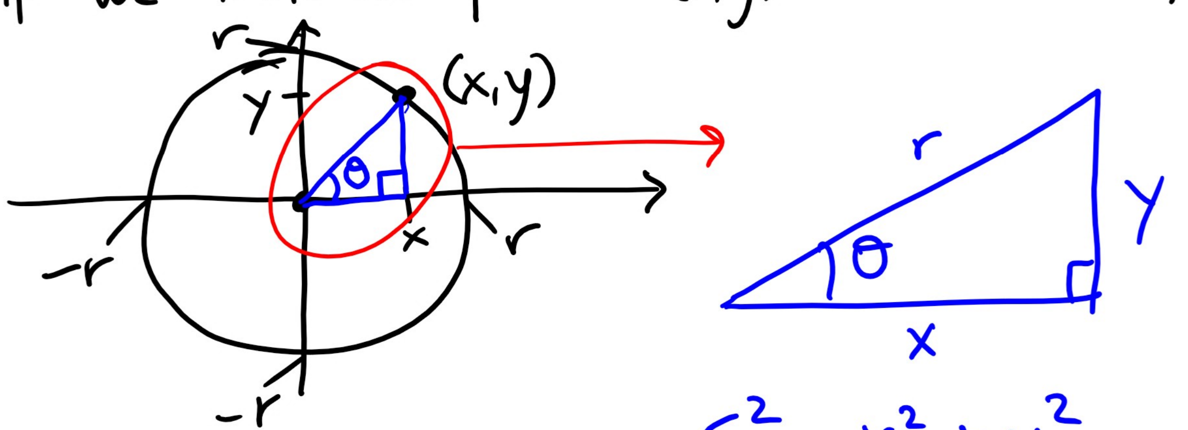


$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \qquad \sin\frac{\pi}{6} = 1/2$$

$$\cos\frac{\pi}{3} = 1/2 \qquad \cos\pi/6 = \sqrt{3}/2$$

$$\tan\pi/3 = \sqrt{3} \qquad \tan\pi/6 = 1/\sqrt{3}.$$

Notice, if we look at point (x,y) on a circle of radius r



$$r^2 = x^2 + y^2$$

from above: $\left.\begin{aligned} y &= r\sin\theta \\ x &= r\cos\theta \end{aligned}\right\} \longrightarrow$ so $\cancel{r^2} = \cancel{r^2}\sin^2\theta + \cancel{r^2}\cos^2\theta$

Trig. Identity $\longrightarrow$ $1 = \sin^2\theta + \cos^2\theta$

We can also see $\tan\theta = y/x = \dfrac{\cancel{r}\sin\theta}{\cancel{r}\cos\theta} = \dfrac{\sin\theta}{\cos\theta}.$

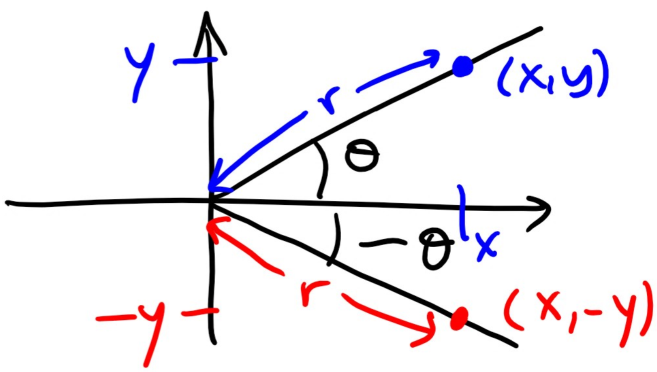
More trig. identities:

from $\sin^2\theta + \cos^2\theta = 1$, divide by $\cos^2\theta$:

$$\frac{\sin^2\theta}{\cos^2\theta} + 1 = \sec^2\theta$$

i.e. $\underline{\tan^2\theta + 1 = \sec^2\theta.}$ ← This one is very, very useful. Remember it!!!

Or divide by $\sin^2\theta$: $\underline{1 + \cot^2\theta = \csc^2\theta}$



$\sin\theta = y/r$ $\quad$ $\cos\theta = x/r$

$\sin(-\theta) = \frac{-y}{r}$ $\quad$ $\cos(-\theta) = x/r$

$\sin\theta = -\sin(-\theta)$

Sin is an odd function

(odd f: $f(x) = -f(-x)$)

$\cos\theta = \cos(-\theta)$

cos is an even function

(even f: $f(x) = f(-x)$)