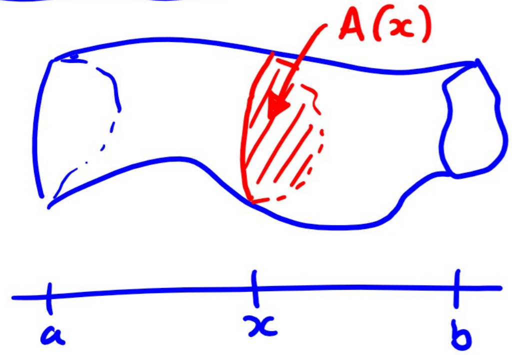
1ZA3 (SECTION C01) – ENGINEERING MATHEMATICS I

Lecture 29

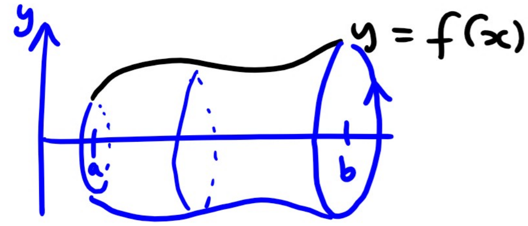
Last time

VOLUMES

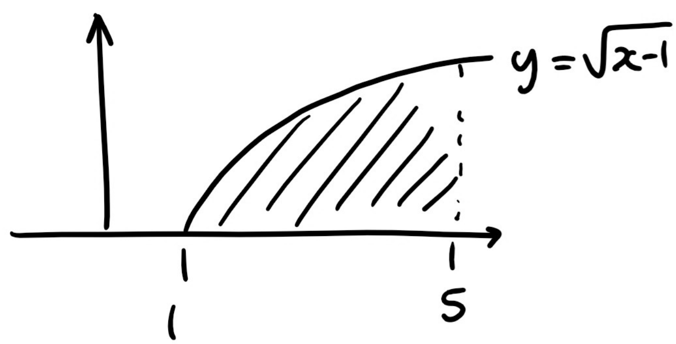
$$\text{Volume of solid} = \int_a^b A(x)\,dx$$

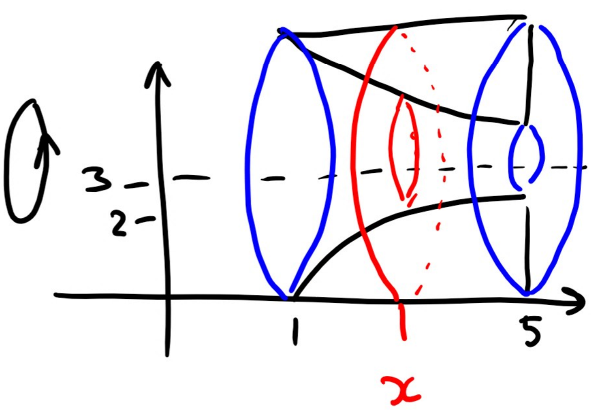


$$\text{Volume of a solid of revolution} = \int_a^b \pi (f(x))^2\,dx$$



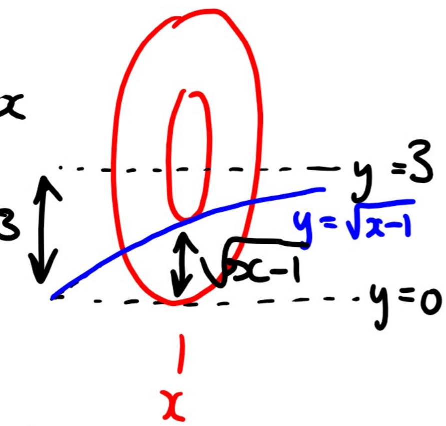
Example $y = \sqrt{x-1}$ on $[1,5]$ rotated around $y=3$





Cross-section at x

$A(x) =$ area of outer circle − area of inner circle



$$= \pi 3^2 - \pi\left(3 - \sqrt{x-1}\right)^2$$

$$\text{Volume} = \pi \int_1^5 9 - \left(3 - \sqrt{x-1}\right)^2 dx = \pi \int_1^5 6\sqrt{x-1} - x + 1\ dx$$

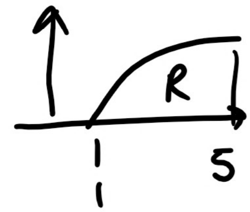
$$= \dots = \pi \left[4(x-1)^{3/2} - \frac{x^2}{2} + x \right]_1^5$$

Substitution Rule... (Check!)

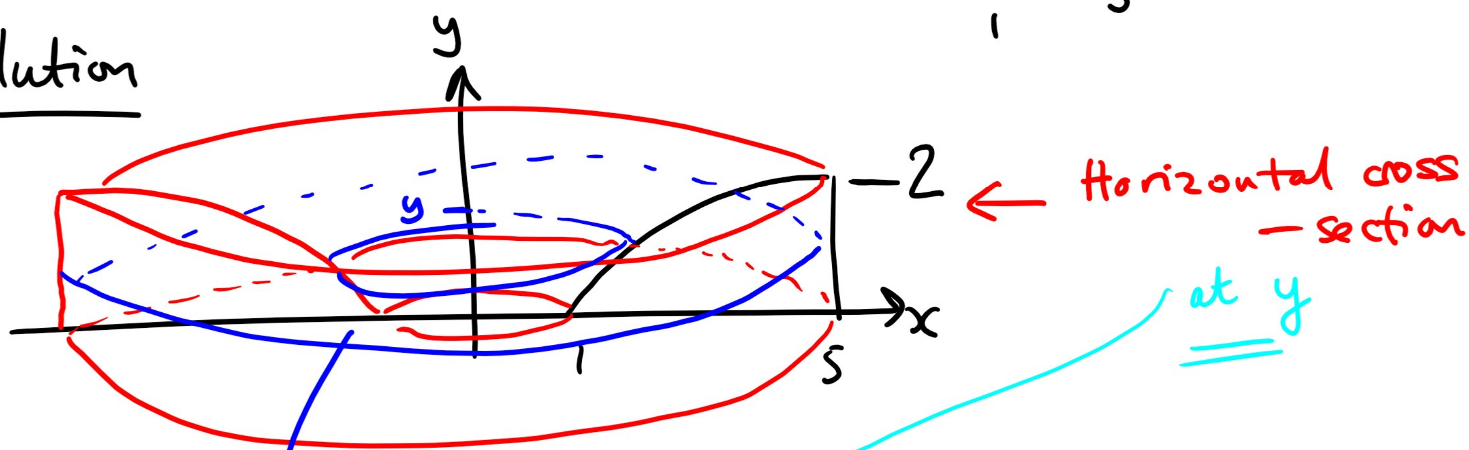
$$= \dots = \underline{\underline{24\pi}}$$

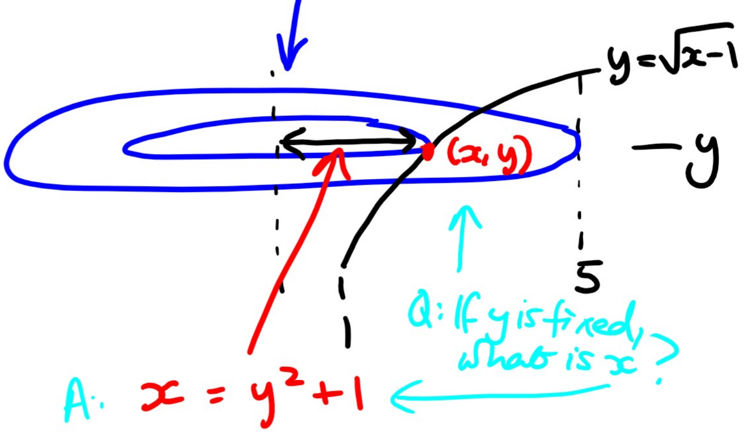
(Check!)

<u>Example</u> Find volume of solid generated by rotating R around y-axis.



<u>Solution</u>





$A(y)$ = Area of cross-section

= area of outer circle − area of inner circle

$$= \pi 5^2 - \pi (1+y^2)^2$$

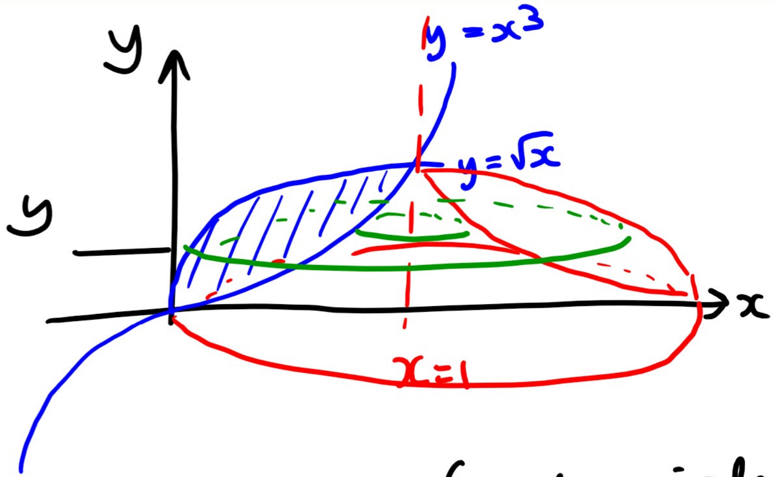
Now integrate <u>over</u> y from $y=0$ to $y=2$

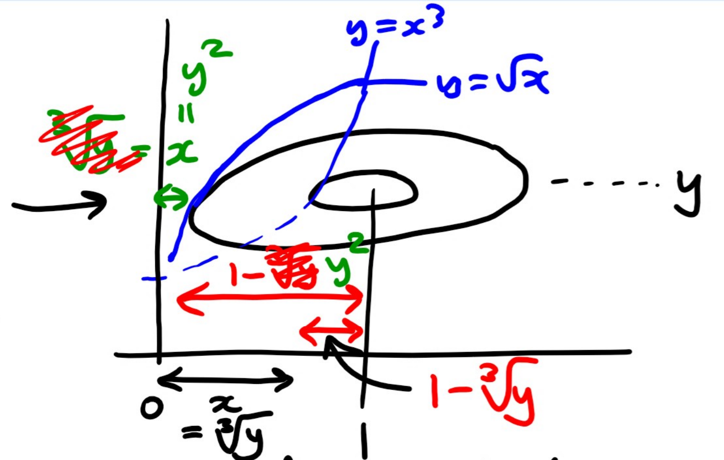
and we get $\pi \int_0^2 25 - (1+y^2)^2 \, dy$

$$= \dots = \frac{544\pi}{15}.$$

<u>Example</u> Find the volume of solid generated by rotating the region bounded by $y=x^3$ & $y=\sqrt{x}$ with $y \geq 0$ around $x=1$.

<u>Solution</u>





$A(y) =$ area of outer circle — area of inner circle

$$= \pi\left(1-\cancel{\sqrt[3]{y}}\;y^2\right)^2 - \pi\left(1-\sqrt[3]{y}\right)^2$$

Now integrate from $y=0$ to $y=1$.

$$\int_0^1 A(y)\,dy = \int_0^1 \pi\left(\left(1-y^2\right)^2 - \left(1-\sqrt[3]{y}\right)^2\right) dy.$$

6.4 WORK

Goal: Find work needed to move an object along a straight line from a to b.

$$\text{Work} = \underbrace{\text{Force}}_{= \text{Mass} \times \text{Accel.}} \times \text{Distance}$$

Units	Force	Work
Metric	$kg\,m/s^2 = N$ (Newton)	$Nm = J$ (Joule)
Imperial	lb (pounds)	ft-lb

Example How much work is done to lift a 15kg armadillo 2m off the ground?

Solution Force needs to counter gravity $g = 9.8\,m/s^2$

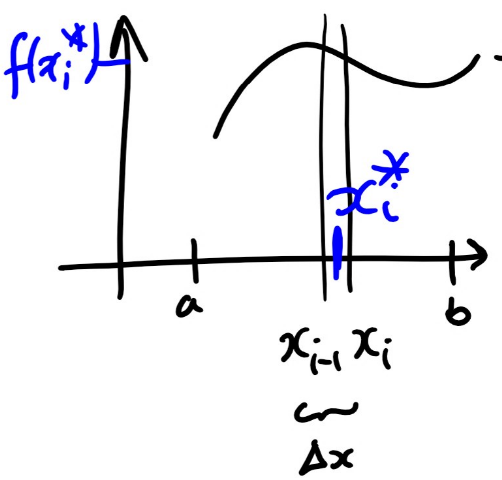
$$W = \text{Work} = \text{Force} \times \text{distance} = (\overset{m}{15} \times \overset{a}{9.8}) \times 2 = \underline{\underline{294J}}.$$

But what do we do if force depends on distance?

e.g. **Hooke's Law** Force required to maintain a spring x units beyond its natural length  is proportional to x.

So force $= kx$ (some k constant)

If we have a function $f(x)$ for force, then:



- Divide up $[a,b]$ into n subintervals $[x_{i-1}, x_i]$ of equal length $\Delta x = \frac{b-a}{n}$
- If x_i^* is a sample point in $[x_{i-1}, x_i]$ then $f(x_i^*)$ approximates $f(x)$ on $[x_{i-1}, x_i]$.
- So work done moving object from x_{i-1} to x_i is force $\times$ distance $\approx f(x_i^*) \cdot \Delta x$

- Total work $\approx \sum_{i=1}^{n} f(x_i^*) \Delta x$ $= ($work to get from a to x_1 + " " " " x_1 to x_2 + " " " " x_2 to x_3 + ... + " " " " x_{n-1} to b $)$

- Total work $= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*) \Delta x = \int_a^b f(x)\,dx.$

<u>Example</u> Suppose a force of 10N is required to stretch a spring 5 cm from natural length.
How much work is done in stretching it 15cm?

<u>Solution</u> $f(x) = kx$ for some k by Hooke's Law.

So find k: $f(\underbrace{0.05}_{5\text{cm}}) = 10$

So $(0.05)k = 10 \Rightarrow k = \frac{10}{0.05} = 200.$

So $f(x) = 200x$.

Total work $W = \int_0^{\overbrace{0.15}^{15\text{cm}}} 200x\,dx = \left[100x^2\right]_0^{0.15} = \underline{\underline{2.25J}}$