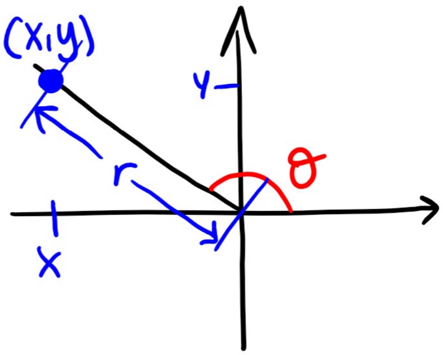
1ZA3 (SECTION C01) – ENGINEERING MATHEMATICS I

Lecture 3

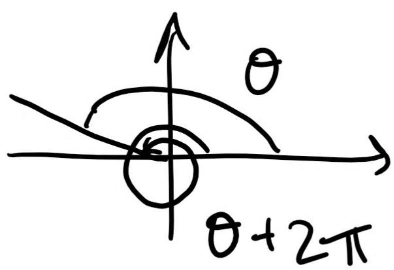
Last time: Trigonometry



$$\sin\theta = \frac{y}{r} \qquad \tan\theta = \frac{y}{x}.$$

$$\cos\theta = \frac{x}{r}$$

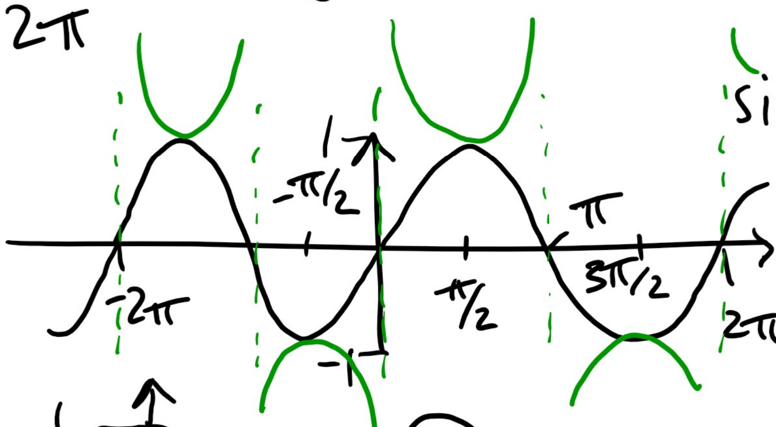
What if $x = 0$?
Or $y = 0$?



so $\sin(\theta) = \sin(\theta + 2\pi)$

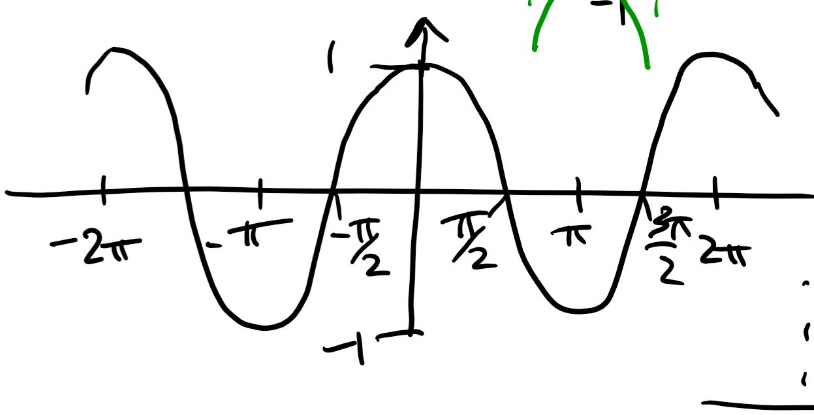
etc.

Graphs



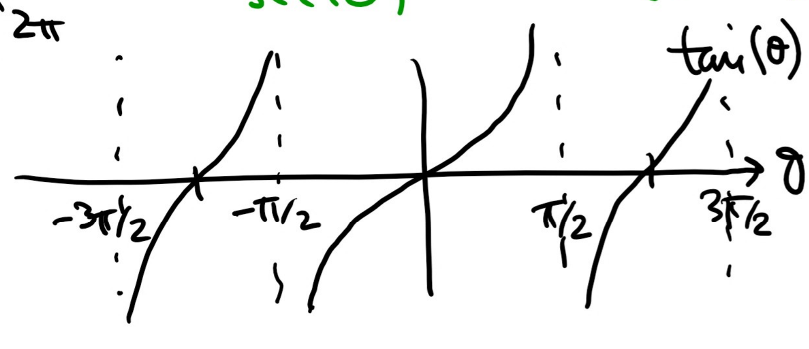
sin(θ)
csc(θ)

odd



cos(θ)
sec(θ)

[cot(θ)]



One more useful (set of) trig. identitylies

Addition Formulae $\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$

In particular $\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\left(= 1 - 2\sin^2\theta = 2\cos^2\theta - 1\right)$

Half-angle formulae come from these too ...

1.5 Inverse Functions & Logarithms

$y = f(x)$

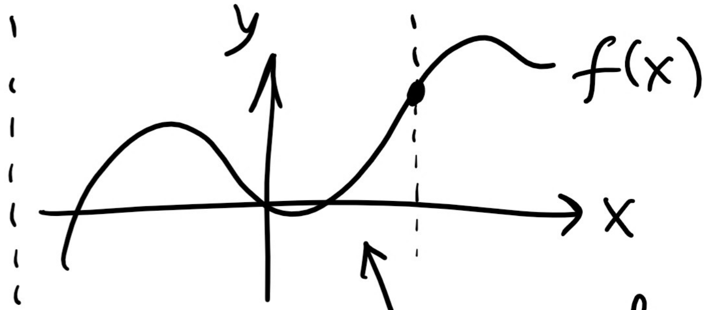
f - function

x - "input" independent variable

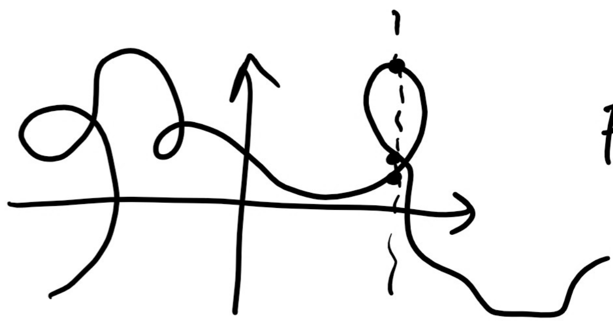
y - "output" dependent variable

collection of useable inputs x that we can plug into f is the domain of f $\mathrm{dom}(f)$

collection of all outputs ever produced by f is the range of f $\mathrm{ran}(f)$



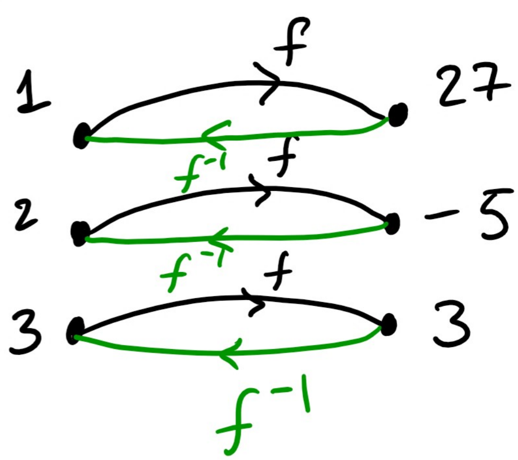
graph of a function because it passes the Vertical Line Test (VLT) i.e. any vertical line crosses the line at most once



FAILs VLT.

Inverse Functions

: "undoing" functions
functions "reversed"



$f(1) = 27$ etc.

$\mathrm{dom}(f) = \{1, 2, 3\}$ curly $\{\ \}$; collection containing 1, 2, 3

$\mathrm{ran}(f) = \{27, -5, 3\}$

f^{-1} reverses f : it's called the <u>inverse</u> of f

$$\mathrm{dom}(f^{-1}) = \{27, -5, 3\} = \mathrm{ran}(f)$$

$$\mathrm{ran}(f^{-1}) = \mathrm{dom}(f).$$

What is $f^{-1}(f(x))$ for any $x \in \operatorname{dom}(f)$?

$= x$

"belongs to"
"in"

Similarly $\underbrace{f(f^{-1}(x))} = x$ for any $x \in \operatorname{dom}(f^{-1})$.

i.e. f undoes f^{-1} i.e. f is the inverse of f^{-1}

"f inverse"

Example Find $f^{-1}(x)$ when $f(x) = \frac{1}{2+x}$.

!!! x !!!

Solution

Step 1 $y = \frac{1}{2+x}$

Step 2 Solve for x:

$$(2+x)y = 1$$

$$2y + xy = 1$$

$$xy = 1 - 2y$$

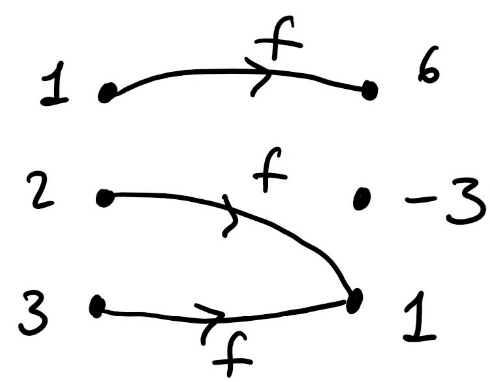
$$x = \frac{1-2y}{y}.$$

This is $f^{-1}(y)$.

But we're asked for $f^{-1}(x)$

Step 3 Change the name of the input variable from y to x i.e. $f^{-1}(x) = \frac{1-2x}{x}$.

New Example



What is $f^{-1}(x)$ for $x \in \text{ran}(f)$?
$= \{6, 1\}$

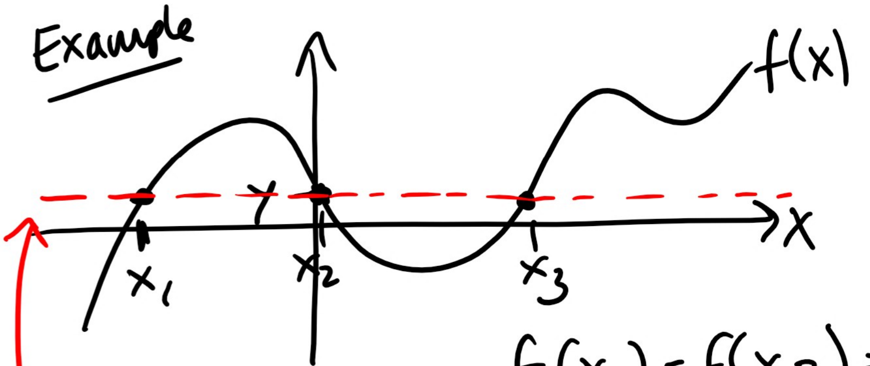
$f^{-1}(6) = 1$. But $f^{-1}(1) = 2$? 3 ?

This f does NOT have an inverse !!!

f^{-1} meaningless ! Does not exist !

When does a function have an inverse ?

Example



We need to rule out that f sends two different inputs to the same output

$f(x_1) = f(x_2) = f(x_3) = y$. Oh dear.

Line hits more than one point on the graph. Fails HLT:

Horizontal Line Test: every horizontal line hits graph at most once.

Definition A function is <u>one-to-one</u> (1-1) when it passes the HLT i.e. no two inputs are sent to the same output i.e.

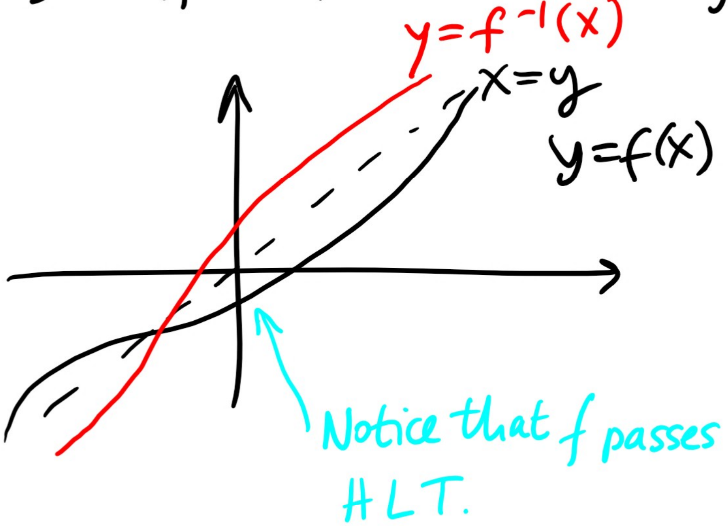
if $f(x_1)=f(x_2)$ then $x_1=x_2$.

A function has an inverse if it is 1–1!!!

<u>If</u> f^{-1} can be defined, what does its graph look like?

If $f(a)=b$, then $f^{-1}(b)=a$

so if (a,b) is on the graph of f, then (b,a) is on the graph of f^{-1}!



graph(f^{-1}) is reflection in line $y=x$ of graph(f).