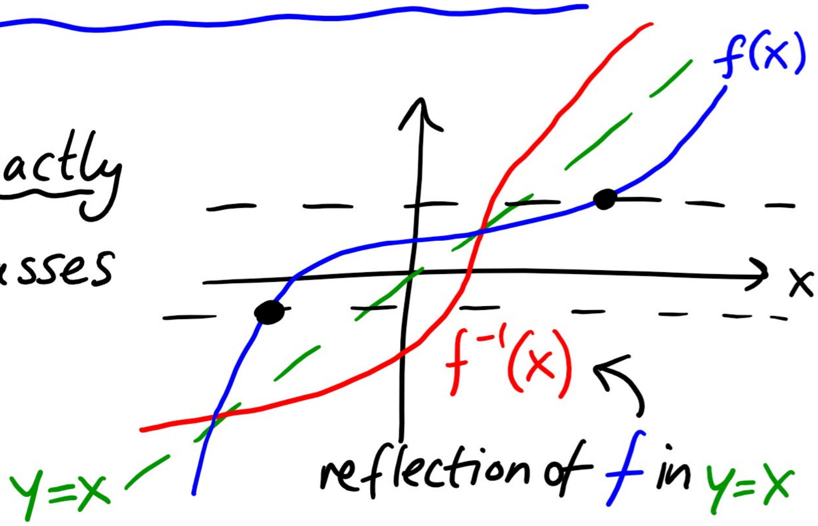
# 1ZA3 (SECTION C01) – ENGINEERING MATHEMATICS I

Lecture 4

**Last time** — Inverse and 1 – 1 Functions

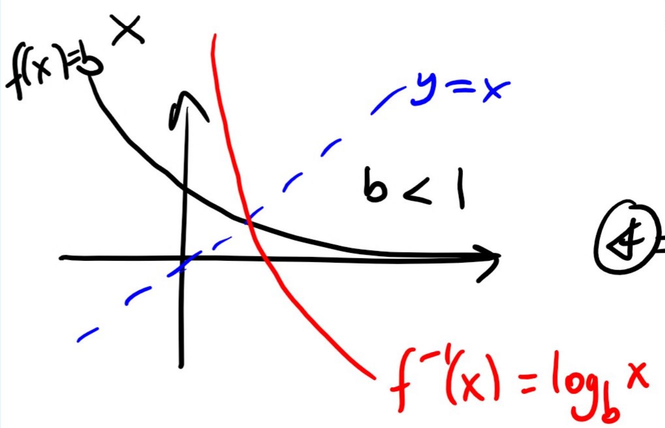
$f$ has an inverse $f^{-1}$ exactly when it is 1 - 1 i.e. passes the Horizontal Line Test:

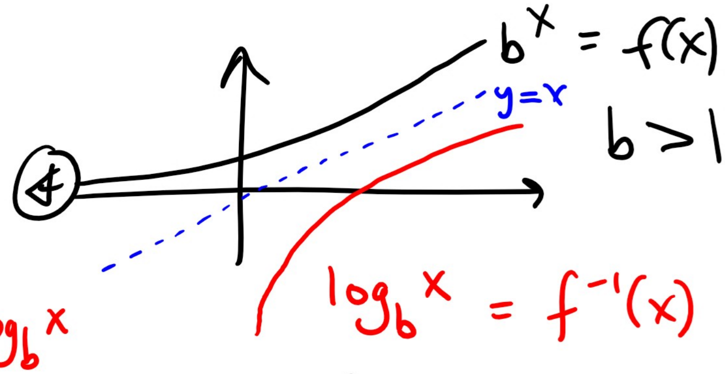



## Logarithmic Functions

$f(x) = b^x$ – exponential function

$b > 0 \quad (b \neq 1)$




exponent to which $b$ must be raised to get $x$ i.e. $b^{\log_b x} = x$

Also $\left[(f^{-1})^{-1} = f\right]$ we get $\log_b(b^x) = x$ }.

Crucial Example   Natural logarithm where

$b = e \approx 2.718\ldots$   $e$ = Euler's constant

$\log_e(x)$ written $\ln(x)$

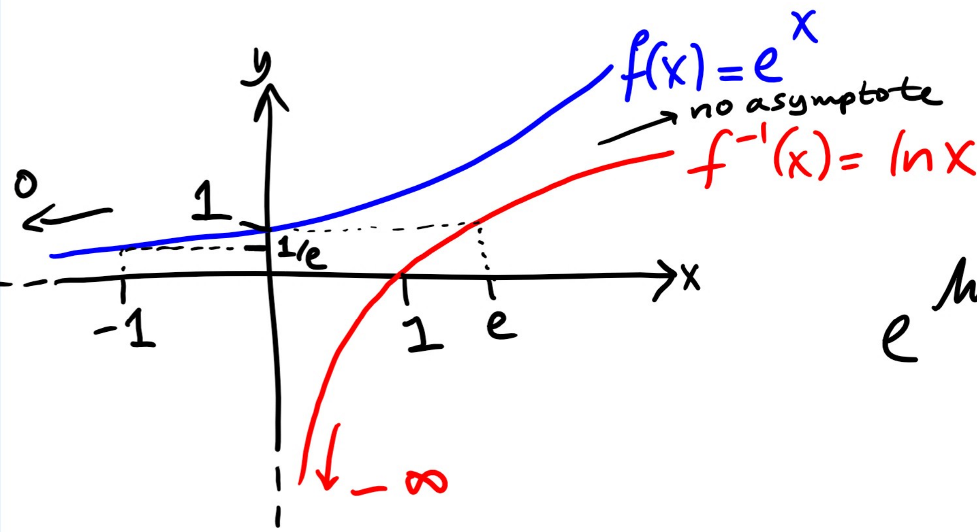
Warning: $\log \neq \log$ !   (Base 10 preferred by engineers, base $e$ by math.)

$(= \log_{10})$   $(= \log_e)$

Change of base formula   For any $b > 0$, $b \neq 1$, any $x \in \mathbb{R}$

↑ real #s

$$\log_b x = \frac{\ln x}{\ln b} = \underbrace{\left(\frac{1}{\ln b}\right)}_{\text{constant multiple}} \ln x$$




So from now on, we'll only typically use ln.

$$e^{\ln x} = \ln(e^x) = x$$

Log Rules

(1) $\ln(xy) = \ln(x) + \ln(y)$

(2) $\ln(x^r) = r\ln(x)$

(3) $\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$

Same goes for any $\log_b$.

**Example** Find $x$ when $\ln(3x) = 7 + \ln(x^2)$.

**Solution**

$$\Rightarrow \ln(3) + \cancel{\ln(x)} = 7 + \cancel{2}\ln(x)$$

$$\Rightarrow \ln(3) - 7 = \ln(x)$$

$$\Rightarrow e^{\ln(3)-7} = e^{\ln(x)} = x$$

$$\Rightarrow e^{\ln(3)} \cdot e^{-7} = x$$

$$\Rightarrow 3e^{-7} = x \Rightarrow \frac{3}{e^7} = x$$

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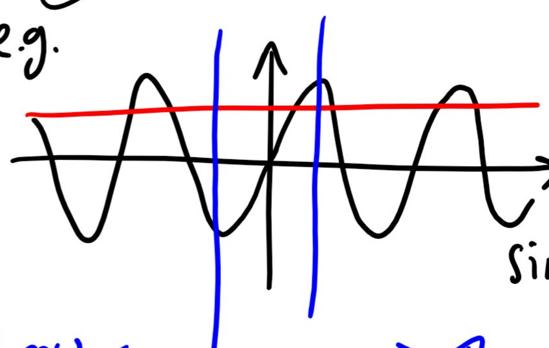
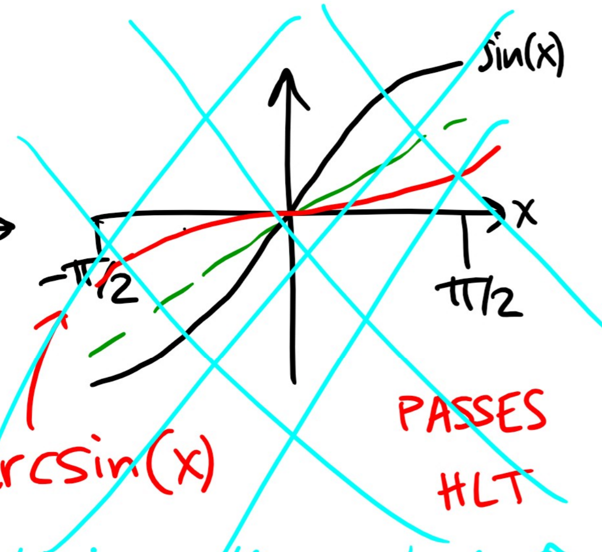
## Inverse Trig. Functions

e.g.

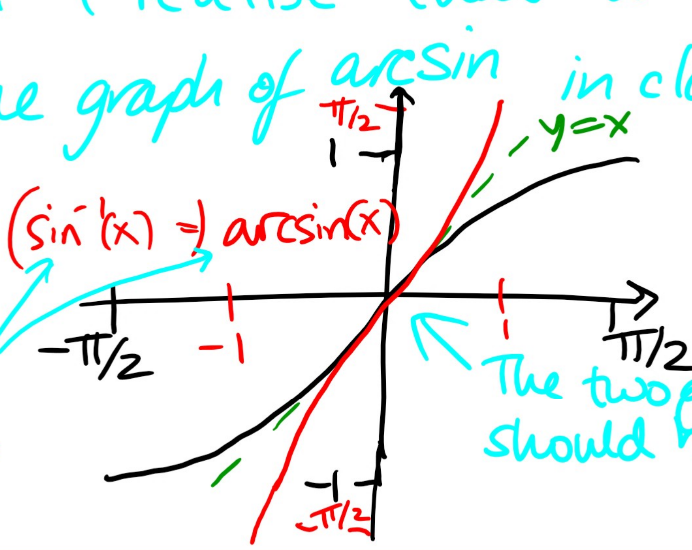



$\sin^{-1}(x) = \arcsin(x)$

Very sorry! I realise that this attempt to draw out the graph of arcsin in class was wrong.

Here it is properly:




sin(x) PASSES HLT

2 names for the same function

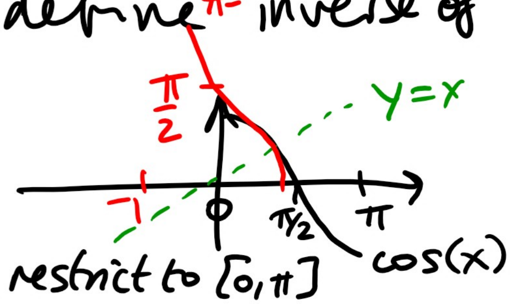
The two graphs should have same slope at 0

As long as $x \in \left[-\pi/2, \frac{\pi}{2}\right]$ i.e. restrict domain

include endpoints

we can define inverse of $\sin(x)$ called $\arcsin(x)$

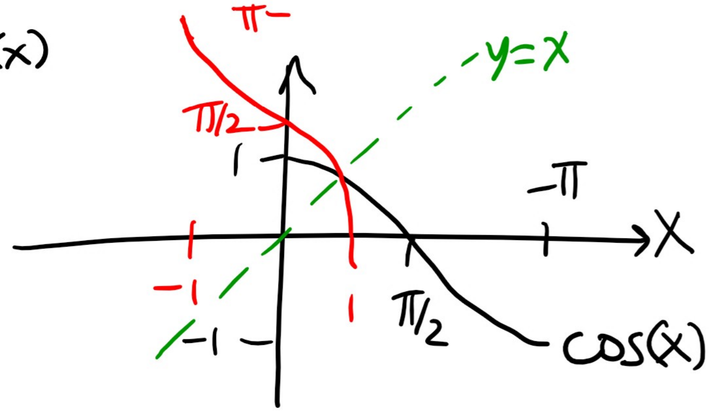
Same idea for $\cos(x)$ :

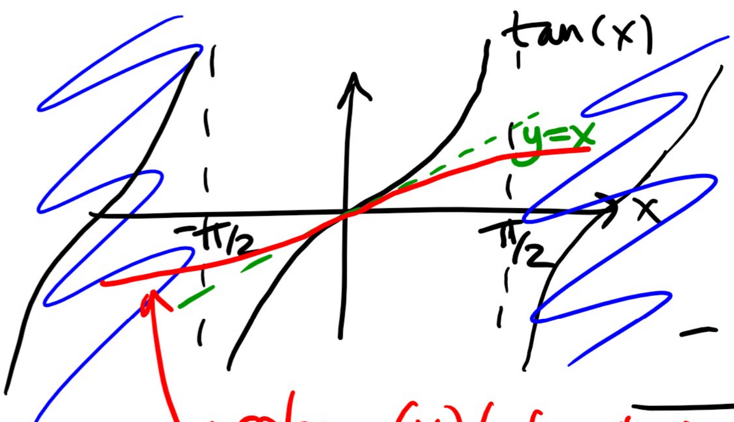



This is OK but not so clear, so here is a better version:

different names for the same function

$\arccos(x)$ $(= \cos^{-1}(x))$



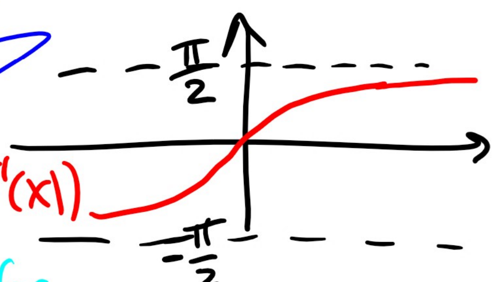




$\arctan(x)$ $(= \tan^{-1}(x))$

different names for the same function

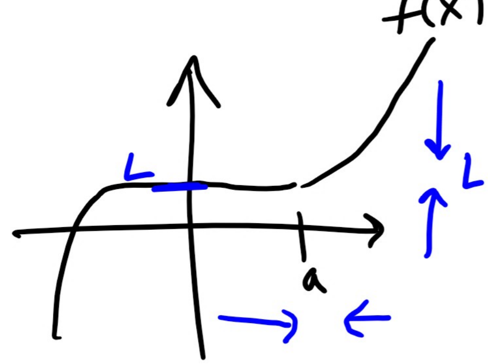
Restrict to $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

not included




General principle: if we can restrict the domain of a function $f$ to one on which graph($f$) passes HLT, then we can define an inverse $f^{-1}$ (for that restricted domain).

# 2.5 Limits & Continuity




$f(x)$ defined near a point $a$

(we don't necessarily know what it does <u>at</u> $a$)

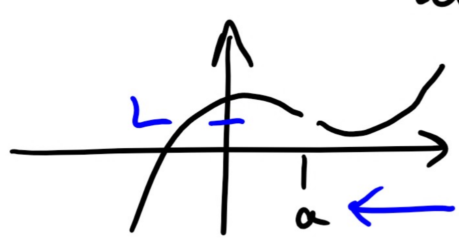
We say the "limit of $f(x)$ as $x$ tends to $a$" is $L$ (a#)

(written $\lim_{x \to a} f(x) = L$ ) if we can make $f(x)$ as close to $L$ as we like by taking $x$ as close as we like to $a$.

Also sometimes written $f(x) \to L$ as $x \to a$ for short.

## One-sided Limits

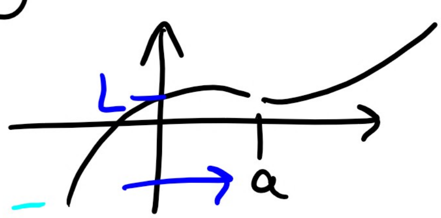
$$\lim_{x \to a^+} f(x) = L$$ ← we only look at $x > a$:




$f(x) \to L$ as $x \to a^+$.

$$\lim_{x \to a^-} f(x) = L$$

we only look at $x < a$:




$f(x) \to L$ as $x \to a^-$