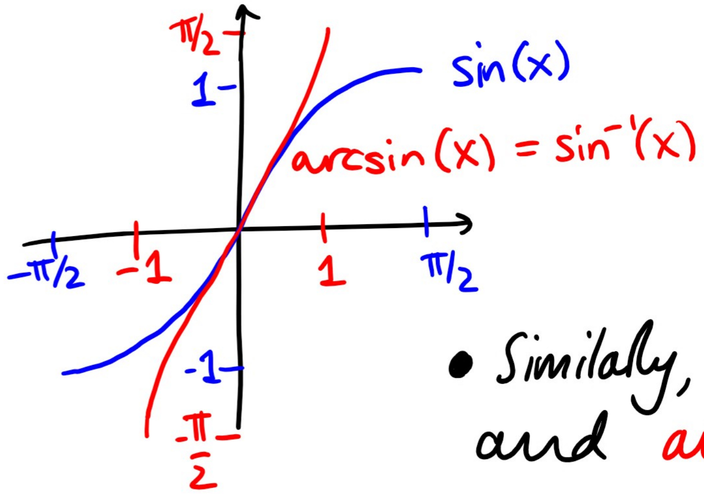
1ZA3 (SECTION C01) – ENGINEERING MATHEMATICS I

Lecture 5

Last time Inverse trigonometric functions



- Restrict sin to $[-\frac{\pi}{2}, \frac{\pi}{2}]$ to be able to define arcsin.
- So arcsin can only take values in $[-\frac{\pi}{2}, \frac{\pi}{2}]$.
- Similarly, arccos can only take values in $[0, \pi]$ and arctan can only take values in $(-\frac{\pi}{2}, \frac{\pi}{2})$.

Example $\arcsin(\sin(\frac{3\pi}{2})) = \arcsin(-1) = -\frac{\pi}{2}$

Example Simplify $\sin(\underbrace{\arctan(x)})$.

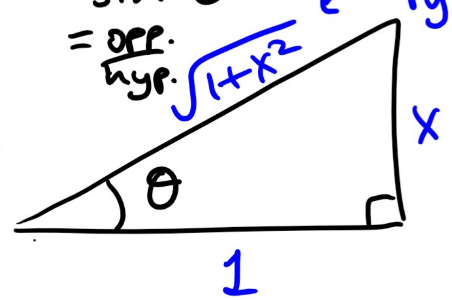
Solution Name this e.g. $\theta = \arctan(x)$

θ has to be in $(-\frac{\pi}{2}, \frac{\pi}{2})$

What we're asked to do is find an expression in terms of x for $\sin\theta$ $= \frac{\text{opp.}}{\text{hyp.}}$

We know $\tan\theta = x = \frac{x}{1}$ $= \frac{\text{opp.}}{\text{adj.}}$

use Pythagoras!

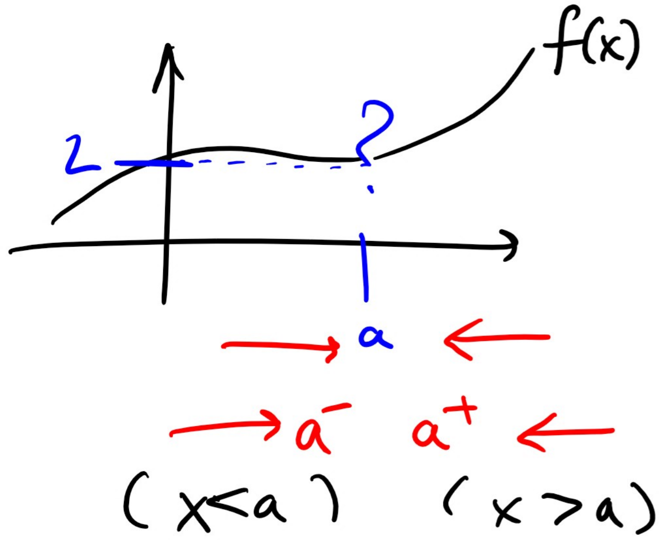


So $\sin\theta = \boxed{\frac{x}{\sqrt{1+x^2}} = \sin(\arctan(x))}$.

Back to Limits

Last time:

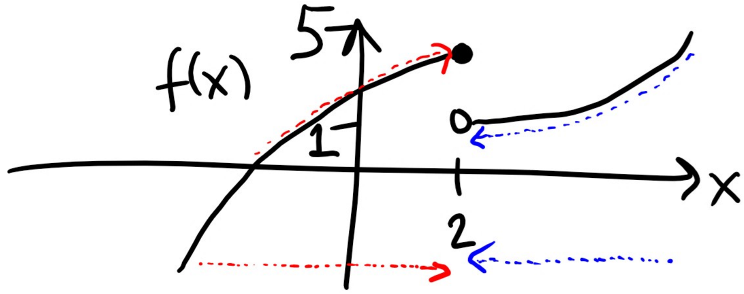
$$\lim_{x \to a} f(x) = L$$



$$\lim_{x \to a^-} f(x) = L$$

$$\lim_{x \to a^+} f(x) = L$$

Example



Here $\lim_{x \to 2^-} f(x) = 5$ $\quad \lim_{x \to 2^+} f(x) = 1$ even though $f(2)=5$

$\lim_{x \to 2} f(x) = \text{DNE}$ (does not exist, undefined)

as $\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$

In general, $\lim_{x \to a} f(x) = L$ (a #) exactly when $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$

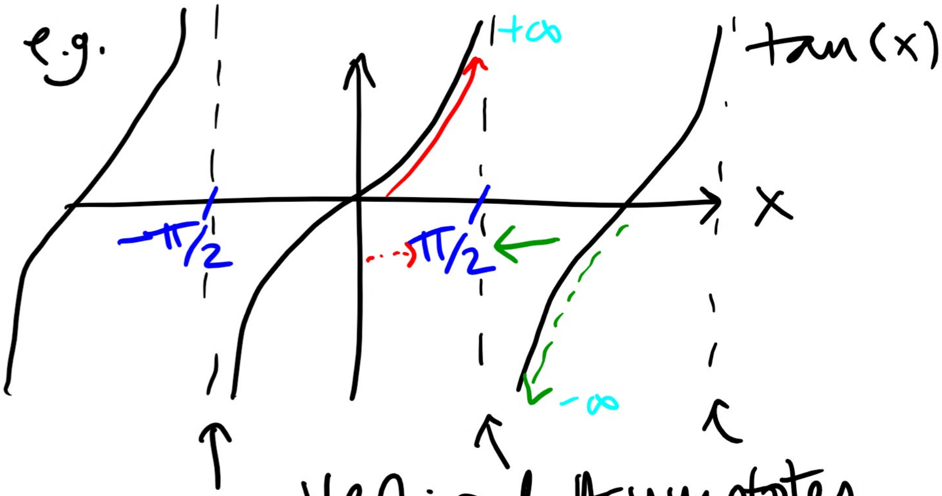
Infinite Limits

$\lim_{x \to a} f(x) = \pm\infty$ means

[Not really =]

we can make the value of $f(x)$ as big ($+\infty$) or as small ($-\infty$) as we like by taking x as close to a as we like.

e.g.



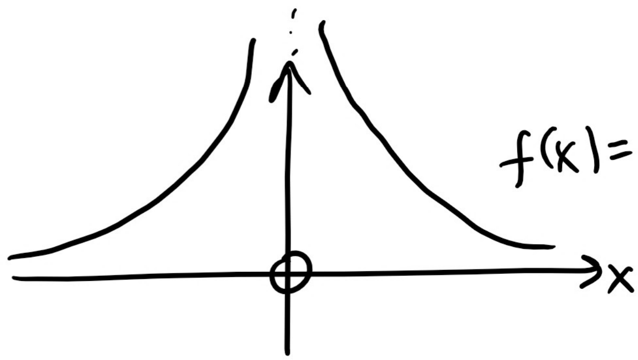
Vertical Asymptotes

$$\lim_{x \to \frac{\pi}{2}^-} \tan(x) = \infty$$

$$\lim_{x \to \frac{\pi}{2}^+} \tan(x) = -\infty$$

$$\lim_{x \to \pi/2} \tan(x) \text{ DNE.}$$

e.g.



$f(x) = \frac{1}{x^2}$

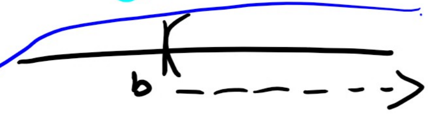
$$\lim_{x \to 0^-} 1/x^2 = \infty \quad \text{DNE}$$

$$\lim_{x \to 0^+} 1/x^2 = \infty \quad \text{DNE}$$

$$\lim_{x \to 0} \frac{1}{x^2} \text{ DNE}$$

$= \infty$ ← "A limit exists" means it equals a number.

Knowing the limit "is ±∞" tells us in what way the limit DNE.



Limits at Infinity

If $f(x)$ defined on (b, ∞) for some b (or $(-\infty, b)$)

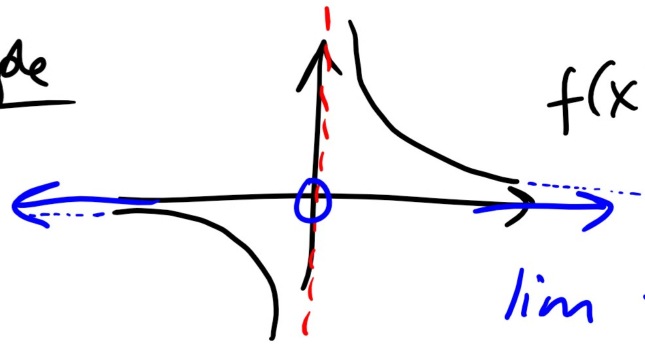
e.g. $x > 3$

we can talk about $\lim_{x \to \infty} f(x) = L$ (#) ←

means we can make the value of $f(x)$ as close to L as we like by taking x as big as we like

(or $\lim_{x \to -\infty} f(x) = L$ (#) if .... taking x as small as we like)

Example



$f(x) = \frac{1}{x}$

Watch out: also an infinite limit here! (As mentioned in class, this is left for you to think about.)

$$\lim_{x \to \infty} \frac{1}{x} = 0$$

$$\lim_{x \to -\infty} \frac{1}{x} = 0$$

Remember (?) your Limit Laws

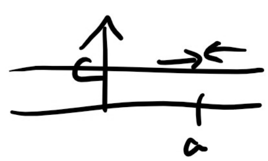
If $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist,

a could be a # or ∞ or $-\infty$ (must be same)

then (a) $$\lim_{x \to a} \left(f(x) \pm g(x)\right) = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$$

(b) $$\lim_{x \to a} \left(f(x)g(x)\right) = \left(\lim_{x \to a} f(x)\right)\left(\lim_{x \to a} g(x)\right)$$

(c) $$\lim_{x \to a} \left(c f(x)\right) = c \lim_{x \to a} f(x)$$



(d) $$\lim_{x \to a} \left(\frac{f(x)}{g(x)}\right) = \left(\lim_{x \to a} f(x)\right) \Big/ \left(\lim_{x \to a} g(x)\right)$$

IF $\lim_{x \to a} g(x) \neq 0$

To reiterate:

DNE means NOT equal to a # ("=" $\pm\infty$ also DNE)