1B03 – LINEAR ALGEBRA 1 (C01) WS19 Lecture 1

Linear Equations (L.E.s)

x, y variables

✓ ① $x + 3y = 6$ $\quad (y = mx + c)$

m, c constants

✓ ② $3x + 2y - z = \sqrt{7}$

$(ax + by + cz = d)$

x, y, z – variables

a, b, c, d – constants

✗ ③ $6x^2 + 5x = 3$

x raised to power > 1

✗ ④ $2x - 6xy + 7y = 0$

x & y should be in terms on their own

✓ ⑤ $17x_1 + \sqrt{3}\,x_2 = \pi x_3 + 2$ – x_1, x_2, x_3 variables

– any variables can be mult. by constants

– constants can appear in terms on their own

✗ ⑥ $e^x + y = 5$

✗ ⑦ $\sin x + 7y - 1 = 0$

✗ ⑧ $x_1 - \sqrt{x_2} = 0$

variables cannot appear as arguments in other functions

The "general form" of a linear equation is

$$a_1x_1 + a_2x_2 + \dots + a_nx_n = b$$

, where the $x_1, \dots, x_n$ are variables, $a_1, \dots, a_n, b$ are constants & a_i is the coefficient of x_i for each i.

As equations, they might (or might not) have solutions : i.e. a list - an <u>n-tuple</u> - of n #s $(s_1, \dots, s_n)$ that satisfies the equation i.e.

$$a_1 s_1 + a_2 s_2 + \dots + a_n s_n = b$$

e.g. $x + 3y = 6$ has as solutions the points on the line $y = -\frac{1}{3}x + 2$

e.g. $(x,y) = (0,2)$ or $(x,y) = (21,-5)$

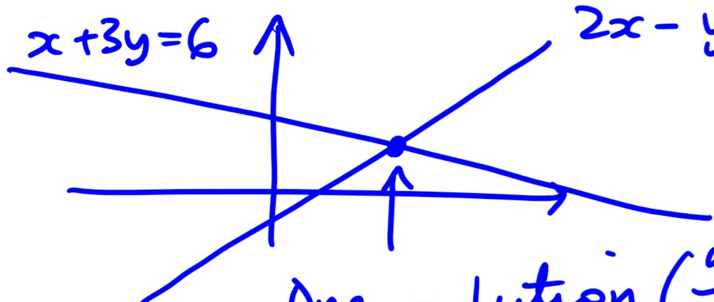
$[0 + 3.2 = 6]$ $[21 + 3(-5) = 6]$.

<u>Systems of Linear Equations</u>

One or more L.E. considered <u>together</u>

- look for solutions to all equations <u>at once</u>.

<u>Example</u> $\left\{ \begin{aligned} x + 3y &= 6 \\ 2x - y &= 1 \end{aligned} \right\}$ Geometrically, solutions of this system are <u>any</u> points of intersection:



<u>One</u> solution $\left(\frac{9}{7}, \frac{11}{7}\right)$

First check:

$\frac{9}{7} + 3\left(\frac{11}{7}\right) = \frac{9}{7} + \frac{33}{7} = 6 \checkmark$

$2\left(\frac{9}{7}\right) - \frac{11}{7} = \frac{18}{7} - \frac{11}{7} = 1 \checkmark$

How did we find the solution $(\frac{9}{7}, \frac{11}{7})$ to

$$\left\{\begin{array}{l} x + 3y = 6 \\ 2x - y = 1 \end{array}\right\} \quad ?$$

Take 2 times the first equation away from 2nd:

Get a new system: $\left\{\begin{array}{l} x + 3y = 6 \\ 2x - y - 2(x+3y) = 1 - 2.6 \end{array}\right\}$

i.e. $\left\{\begin{array}{l} x + 3y = 6 \\ -7y = -11 \end{array}\right\}$

We multiply the second equation by $-\frac{1}{7}$ to isolate y:

$$\left\{\begin{array}{l} x + 3y = 6 \\ y = 11/7 \end{array}\right\}$$

Then solve for x. $x = 6 - 3y = 6 - 3\left(\frac{11}{7}\right) = 6 - \frac{33}{7} = \frac{9}{7}$.

Our first goal in this course is to scale up this strategy (in a way that could also be done computationally) to solve:

General Systems of L.E.s:

$$\left.\begin{array}{ll} 1) & a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ 2) & a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\ \vdots & \quad \vdots \qquad\qquad\qquad\qquad\qquad\qquad \vdots \\ m) & a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m \end{array}\right\} \begin{array}{l} m \text{ equations} \\ \text{in} \\ n \text{ variables} \\ x_1, \ldots, x_n \end{array}$$

$\underbrace{a_{ij}}_{\text{gives location}}$ = coefficient in equation # i of variable x_j.

A solution is an n-tuple $(s_1, \ldots, s_n)$ which is a solution to <u>all</u> m equations at once.