1ZC3 ENGINEERING MATH II-B (Linear Algebra I)

(WS 19 C03) Lecture 1

Linear Equations

√ ① $x + 3y = 6$ $\quad$ ($y = mx + c$ $\quad$ x, y variables, m, c constants

√ ② $3x + 2y - z = \sqrt{7}$ $\quad$ ($ax + by + cz = d$ $\quad$ x, y, z vars $\quad$ a, b, c, d constants)

X ③ $6x^2 + 5x = 3$

power of x is > 1

X ④ $2x - 6xy + 7y = 0$

x & y should appear in terms on their own

√ ⑤ $17x_1 + \sqrt{3}\, x_2 = \pi x_3 + 2$

x_1, x_2, x_3 vars multiplied by constants called <u>coeficients</u>; 2 is <u>constant term</u>

x ⑥ $e^x + y = 5$

x ⑦ $\sin x + 7y - 1 = 0$

x ⑧ $x_1 - \sqrt{x_2} = 0$

} variables cannot appear as arguments of other functions

The "general form" of a linear equation is

$$\boxed{a_1x_1 + a_2x_2 + \dots + a_nx_n = b}$$ where

$x_1, \dots, x_n$ are variables and $a_1, \dots, a_n, b$ are constants.

As equations, these might — or might not — have solution(s) i.e. <u>n-tuple</u> $(s_1, \ldots, s_n)$ that satisfies the equation : $a_1 s_1 + \ldots + a_n s_n = b$

e.g. $x + 3y = 6$ has solutions the points on the line $y = -\frac{1}{3}x + 2$

i.e. $(x,y) = (0, 2)$ or $(x,y) = (21, -5)$

$[0 + 3.2 = 6]$ $\quad$ $[21 + 3(-5) = 6]$

etc.

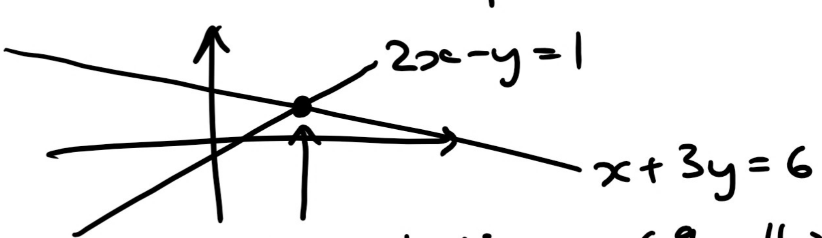
<u>Systems of Linear Equations</u> $\nwarrow$ L.E.S

↳ One or more L.E. considered together.

Look for solutions to all equations at once.

<u>Example</u>
$$\left\{ \begin{aligned} x + 3y &= 6 \\ 2x - y &= 1 \end{aligned} \right\}$$

Geometrically, solutions of this system are <u>any</u> points of intersection.



1 solution $= \left(\frac{9}{7}, \frac{11}{7}\right)$ $\swarrow$ Check :

$\frac{9}{7} + 3\left(\frac{11}{7}\right) = \frac{42}{7} = 6$ ✓

$2\left(\frac{9}{7}\right) - \frac{11}{7} = \frac{7}{7} = 1$ ✓

↑

How did I find the solution?

Recall: $\left\{\begin{array}{l} x+3y=6 \\ 2x-y=1 \end{array}\right\}$

Take 2 times the first equation away from the second:

Get a new system: $\left\{\begin{array}{l} x+3y=6 \\ \underbrace{2x-y-2(x+3y)=1-2.6}_{-7y=-11} \end{array}\right.$

Now multiply the second equation through by $-\frac{1}{7}$

Get $\left\{\begin{array}{l} x+3y=6 \\ y=11/7 \end{array}\right\}$.

Now solve for x in first equation.

$$x = 6-3y = 6-3\left(\tfrac{11}{7}\right) = 9/7.$$

Our first goal is to scale up this strategy — in a way that could also be done computationally — to solve:

General Systems of L.E.s:

m equations in n vars

$$\left\{\begin{array}{lllllll} ① & a_{11}x_1 & +a_{12}x_2 & + & \dots & +a_{1n}x_n & = b_1 \\ ② & a_{21}x_1 & +a_{22}x_2 & + & \dots & +a_{2n}x_n & = b_2 \\ \vdots & \vdots & & & & & \vdots \\ ⓜ & a_{m1}x_1 & +a_{m2}x_2 & + & \dots & +a_{mn}x_n & = b_m \end{array}\right.$$

a_{ij} = coefficient in equation $\#i$ of variable x_j

& a solution is an n-tuple $(s_1, \ldots, s_n)$ which is a solution to all m equations simultaneously.