Name ______________________________

Student Number ______________________________

MATH 2R03 – PRACTICE MIDTERM 2

SUMMER SEMESTER 2018 Dr. Margaret E. M. Thomas
DURATION OF MIDTERM: 1 Hour

THIS TEST PAPER INCLUDES **8** PAGES AND **5** QUESTIONS. IT IS PRINTED ON BOTH SIDES OF THE PAPER. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCIES TO THE ATTENTION OF AN INVIGILATOR.

- Please fill in your name and student number above. You may do this before the test starts.
- Do not open the test paper until the test begins! Once the test starts, please fill in your initials and student number where indicated at the top of each subsequent page.
- Attempt all questions.
- The total number of available points is **30**. Points are indicated next to each question.
- You may use a standard McMaster calculator, Casio FX-991, MS or MS Plus (no communication capability); no other aids are permitted.
- Write your answers in the corresponding spaces provided on the test paper.
- You must show your work to get full credit.
- Two sides at the end are provided for rough work; please ask an invigilator for more rough paper if needed. Please write your student number and initials clearly at the top of each extra page used, and hand in all paper along with your test paper.

Good Luck.

Score

Question	1	2	3	4	5	**Total**
Points	10	6	5	4	5	30
Score						

1. (10 points in total – 2 points for each)
For each of the following statements, state if it is true or false, and provide a VERY BRIEF justification (explanation or counterexample).

   (a) If A is an invertible matrix, then A has a QR decomposition.

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   (b) The transformation $T\colon M_2(\mathbb{R}) \to\colon M_2(\mathbb{R})$ given by $T(A) = A - A^T$ is linear.

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   (c) The space P_5 of real polynomials of degree at most 5 is isomorphic to $M_{3\times 2}(\mathbb{R})$.

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   (d) If $T\colon U \to V$ is a linear transformation with $\text{nullity}(T) = 2$, $\dim(U) = 3$ and $\dim(V) = 4$, then $\text{rank}(T) = 2$.

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   (e) In an inner product space V with $\dim(V) = n$, any orthogonal set of n vectors is a basis for V.

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2. (6 points in total – 2 points for each)
For each part, circle ALL the number(s) corresponding to possible correct way(s) to complete each sentence.

(a) In a QR decomposition of a matrix A,
(1) R is invertible;
(2) Q is square;
(3) R is lower triangular;
(4) R satisfies $R^T R = 0$;
(5) Q has orthogonal column vectors.

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(b) For a subspace W of a vector space V and a vector u in V,
(1) $\langle \text{proj}_W u, u \rangle = 0$;
(2) $||\text{proj}_W u|| = 1$;
(3) $\langle \text{proj}_{W^\perp} u, u \rangle = 0$;
(4) $||u - w|| \geq ||u-$ $\text{proj}_W u||$ for any vector w in W;
(5) $\langle \text{proj}_W u,\ \text{proj}_{W^\perp} u \rangle = 0$.

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(c) If a linear transformation $T\colon U \to V$ is an isomorphism, then
(1) $\ker(T) = \{0\}$;
(2) U = V;
(3) $T(x) = T(y)$ implies $x = y$, for vectors x and y in U;
(4) $\dim(U) = \dim(V)$;
(5) $R(T) = V$.

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3. (5 points) In the space P_3 of real polynomials of degree at most 3 with the inner product given by $\langle a_0+a_1x+a_2x^2+a_3x^3, b_0+b_1x+b_2x^2+b_3x^3\rangle = a_0b_0+a_1b_1+a_2b_2+a_3b_3$, use the Gram–Schmidt Process to find an orthonormal basis for the subspace spanned by $\{x^2-1, x+2, x^3-\sqrt{3}x\}$.

4. (4 points) In the space $M_{2\times 3}(\mathbb{R})$ with the standard inner product $\langle A, B\rangle = \operatorname{tr}(B^T A)$, write

$$C = \begin{pmatrix} -5 & -4 & 2 \\ 1 & -3 & 0 \end{pmatrix}$$

as $C = D+E$, where D lies in $W = \operatorname{span}\left\{ \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 3 & -5 \\ 4 & 2 & 0 \end{pmatrix} \right\}$ and E lies in $W^{\perp}$.

5. (5 points) If U, V and W are vector spaces such that U is isomorphic to V, and V is isomorphic to W, show (just using definitions) that U is isomorphic to W.

ROUGH WORK

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THE END