3Y03 - 3J04 PROBABILITY & STATISTICS FOR (CIVIL) ENGINEERING

C01 - Lecture 11

Last time — POISSON DISTRIBUTION

X - discrete random variable

- counts # "incidents" in given time period/area T
- average # per unit time/area $= \lambda$
- p.m.f. $f(x) = \frac{(\lambda T)^x}{x!} e^{-\lambda T}$.

Example Calls come in to an office at an average rate of 5 per hour. ($5 = \lambda$)

Let X = # calls in 3 hour period. (3 hour $= T$)

Find μ, $P(X=7)$, $P(X \geq 1)$.

Solution

$$\mu = \lambda T = 15.$$

$$P(X=7) = f(7) = \frac{(\lambda T)^7}{7!} e^{-\lambda T}$$

$$= \frac{15^7}{7!} e^{-15}$$

$$= 0.0104.$$

$$P(X \geq 1) = 1 - P(X=0) = 1 - \frac{(\lambda T)^0}{0!} e^{-\lambda T}$$

$$= 1 - e^{-15} = 0.9999\ldots$$

Example: Disc surface contaminated with an average of $\lambda = 0.3$ particles per cm^2 of surface. Disc surface area is $30\ cm^2$.

Find μ, $P(X \leq 2)$. $\leftarrow$ $X = \#$ particles

Solution: $\mu = \lambda T = 0.3 \times 30$ (make sure units match!)

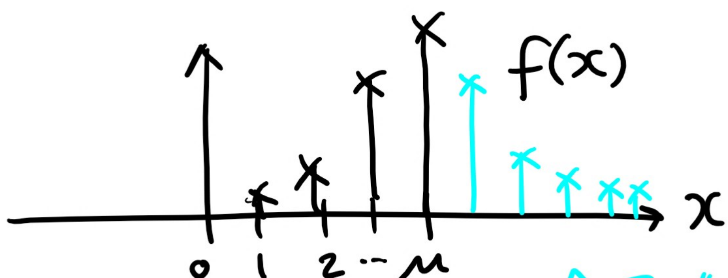
$= 9.$

(Here cm^2; in previous example it was hours.)

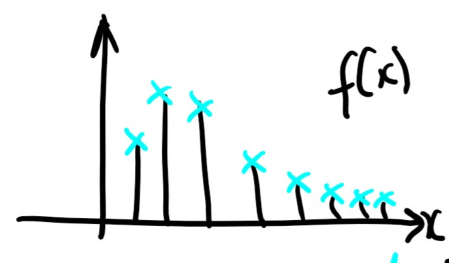
$$P(X \leq 2) = \sum_{x=0}^{2} \frac{9^x}{x!} e^{-9}$$

$$= e^{-9}\left(1 + 9 + \frac{9^2}{2!}\right) = 0.0062.$$

Shape



or



↖ The "curves" I drew first should not be there in discrete r.v. setting.

Chapter 4 Continuous Random Variables

X = some measurement/quantity varying randomly, taking values in real #s or an interval.

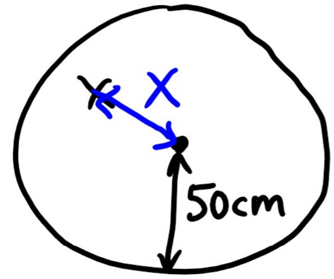
e.g. — time until machine fails
— exact length of manufactured part
— electric current in a wire
— measurement error of a test instrument
— net weight of some packaged item

Even though we could measure to a fixed # decimal places & think "discretely", it's more convenient to imagine all real #s in some range as possible.

<u>Example</u> Throw darts at a circular board

X

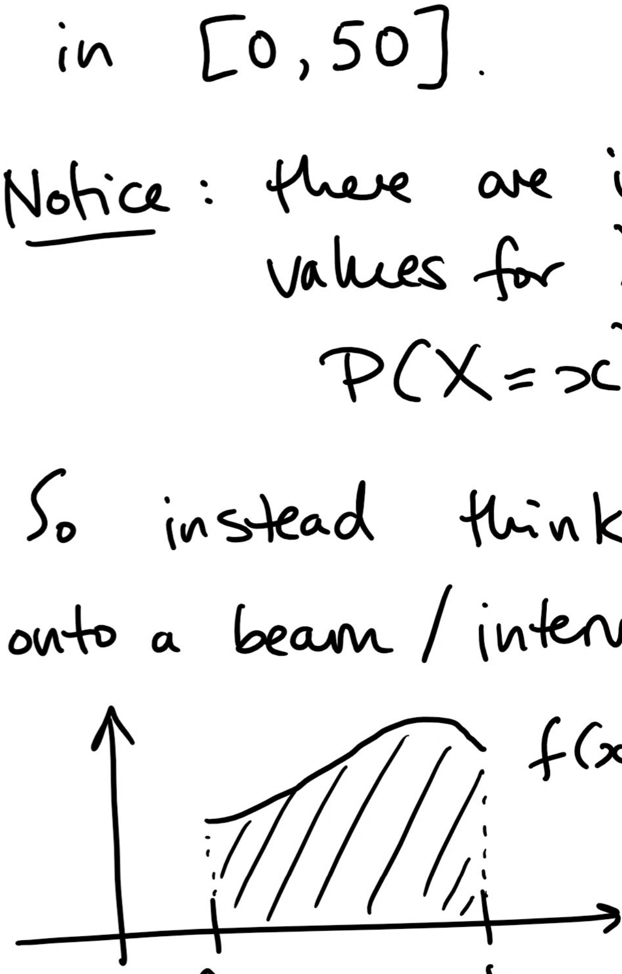
Measure distance from centre (radius). Dartboard radius is 50cm. X can take any value in $[0, 50]$.



<u>Notice</u>: there are infinitely many possible values for X so for a given radius x

$$P(X = x) = \frac{1}{\infty} = 0.$$

So instead think of probability being "loaded" onto a beam / interval & we consider density



$f(x)$ ← density function

Total loading between a and b is given by ...

$$\ldots \int_a^b f(x)\,dx \; .$$ = total loading between a & b.

We define a probability density function (pdf) $f(x)$ of a continuous random variable X to be a function with

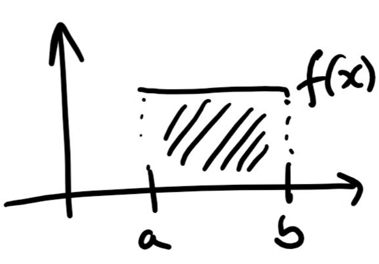
(1) $0 \le f(x)$ $(< \infty)$ ← The point here is that $f(x)$ is NOT bounded by 1 — what's bounded by 1 is the area under the graph

(2) $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx = 1$ ("total of probabilities")

(3) $\displaystyle P(a \le X \le b) = \int_a^b f(x)\,dx \; .$

$\le$ or $<$, $\le$ or $<$ ← Because of the integral, in the continuous case it doesn't matter if we use $\le$ or $<$.

Examples (Uniform Distribution)



$$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{if } a < x < b \\ 0 & \text{otherwise} \end{cases}$$

Check on (2): $\displaystyle\int_a^b \frac{1}{b-a}\,dx = \left[\frac{x}{b-a}\right]_a^b = \frac{b-a}{b-a} = 1$

Say electric current X in wire with possible values between 3.6 and 4.1. We assume this is a — uniform random variable, so has pdf

$$f(x) = \begin{cases} 2 \;\; \left(= \frac{1}{4.1-3.6}\right) & \text{if } 3.6 < x < 4.1 \\ 0 & \text{otherwise.} \end{cases}$$

For example we can calculate:

$$P(3.7 < X < 3.8) = \int_{3.7}^{3.8} 2\,dx = \Big[2x\Big]_{3.7}^{3.8} = 0.2.$$

<u>Example</u> Waiting time X in hours at a hospital for admission is given by distribution with pdf

$$f(x) = \begin{cases} 0.5\,e^{-0.5x} & x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

(Exponential Distribution)

Find $P(1 < X < 2)$ and $P(X > 3)$.

<u>Solution</u>

$$P(1 < X < 2) = \int_{1}^{2} 0.5\,e^{-0.5x}\,dx$$

$$= \Big[-e^{-0.5x}\Big]_{1}^{2}$$

$$= -(e^{-1}) - (-e^{-0.5})$$

$$= e^{-0.5} - e^{-1} = 0.2386.$$

$$P(X>3) = 1 - P(0<X<3)$$

$$= 1 - \int_0^3 0.5e^{-0.5x}\,dx$$

$$= 1 - \left[-e^{-0.5x}\right]_0^3 = 0.2231.$$

CUT OFF FOR TEST 1

4.3) We know $\int_{-\infty}^{\infty} f(x)\,dx = 1$ ← for any pdf $f(x)$.

So in example above $P(X>3)$

$$= 1 - P(0<X<3)$$

$$= \int_{-\infty}^{\infty} 0.5e^{-0.5x}\,dx - \int_0^3 0.5e^{-0.5x}\,dx$$

We can replace the $-\infty$ by 0 → as this function is 0 on $x<0$

$$= \int_3^{\infty} 0.5e^{-0.5x}\,dx.$$

→ To Be Continued...