3Y03 – 3J04 PROBABILITY & STATISTICS FOR (CIVIL) ENGINEERING

C01 – Lecture 25

Today HYPOTHESIS TESTING [9–1]

How to use data to check a claim about a parameter.

A <u>statistical hypothesis</u> is some statement about the parameters of one or more populations.

e.g. $\mu = 50$ or $\sigma_1^2 > \sigma_2^2$.

We start with claim

H_0 : <u>null hypothesis</u> → for us this is some equality statement e.g. $\mu = 50$

H_1 : <u>alternative hypothesis</u> (relative to H_0)

→ some statement that contradicts H_0

e.g. if H_0 is $\mu = 50$,

H_1 could be $\mu \neq 50$ [Can be 2-sided

or $\mu > 50$ or

or $\mu < 50$ $\}$ 1-sided.]

Our goal: use data to decide if we should

— accept H_0

or — reject H_0 and accept H_1

i.e. we assume H_0 & reject H_0 if the data suggests we should.

<u>Possible Objectives</u> 1. Test whether or not a parameter value has changed.

2. Test a theory/model (verify or refute).

3. Conformance testing.

To reject H_0, need a compelling reason — we witness something about the data that is <u>very unlikely</u> to happen if H_0 is true.

We need a <u>test statistic</u> e.g. $\bar{X}$, P, S^2 etc.

& <u>critical values</u> (which define (a) <u>critical region(s)</u>)

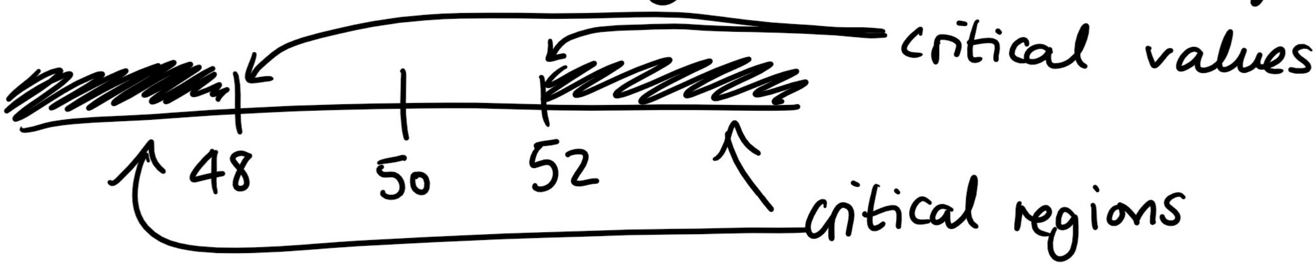
e.g. in testing $H_0 : \mu = 50$

against $H_1 : \mu \neq 50$

we could use as test statistic $\bar{X}$

& we could say reject H_0 & accept H_1 if

$\bar{x} \geq 52$ say or $\bar{x} \leq 48$ say



	H_0 true	H_1 true
Accept H_0	✓ Correct	X Type II Error
Reject H_0 and accept H_1	X Type I Error	✓ Correct

Type I Error : like finding innocent person guilty (under "Presumption of Innocence")

Type II Error : like acquitting a guilty person.

The <u>significance level</u> (or <u>size</u>) of the test

$= \alpha = P(\text{Type I error})$

$= P(\text{rejecting } H_0 \text{ when } H_0 \text{ true}).$

$P(\text{Type II Error}) = P(\text{accepting } H_0 \text{ when } H_1 \text{ true})$

$= \beta$

<u>Example</u> We have sample of size 35 from a Normal population, $\sigma = 10$, to test

$H_0 : \mu = 50 \quad , \quad H_1 : \mu \neq 50.$

We reject H_0 if $\bar{x} \geq 52$ or $\bar{x} \leq 48$.

Find α, and find β if $\mu = 53$.

<u>Solution</u>

$$\alpha = P(\text{Type I error})$$
$$= P(\bar{X} \geq 52 \text{ or } \bar{X} \leq 48 \text{ when } \mu = 50)$$
$$= P\left(Z \geq \frac{52-50}{10/\sqrt{35}} \text{ or } Z \leq \frac{48-50}{10/\sqrt{35}}\right)$$
$$= P(Z \geq 1.18) + P(Z \leq -1.18)$$
$$= (1 - 0.881) + 0.119$$
$$= \underline{\underline{0.238}}.$$

$$\beta = P(48 \leq \bar{X} \leq 52 \text{ when } \mu = 53)$$
$$= P\left(\frac{48-53}{10/\sqrt{35}} \leq Z \leq \frac{52-53}{10/\sqrt{35}}\right)$$
$$= P(-2.96 \leq Z \leq -0.59)$$
$$= 0.277595 - 0.003907$$
$$= \underline{0.274}$$

Want α small. (This is the overriding philosophy: really do NOT want to convict an innocent person.)

<u>Notice</u> : We can make α smaller by increasing n or we can increase the size of the critical region

But increasing size of critical region increases β.

(So better to increase n, as this decreases α and β, if you can — but maybe can't.)

So we <u>fix α</u> in advance.

Usually $0.01 \leq \alpha \leq 0.1$.

Measure of risk & depends on context.

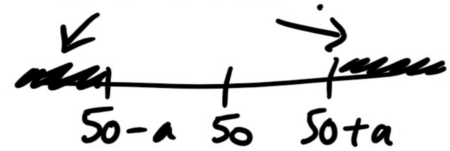
Then determine critical region using α & β. ← Notice β still depends as well on true value of parameters.

<u>Notice</u> As the true value of the parameter being tested approaches the critical values, β approaches $1-\alpha$.

<u>Example</u> (continued) $H_0: \mu = 50$, $H_1: \mu \neq 50$
Normal distribution $\sigma = 10$.

Find the critical region if we want $\alpha = 0.05$.

__Solution__ Call the critical region $50 \pm a$.



We want to find a so that

$$\alpha = 0.05 = P\left(\bar{X} \geq 50 + a \text{ or } \bar{X} \leq 50 - a \text{ when } \mu = 50\right)$$

$$= P\left(\frac{\cancel{50} + a - \cancel{50}}{10/\sqrt{35}} \leq Z\right) + P\left(Z \leq \frac{\cancel{50} - a - \cancel{50}}{10/\sqrt{35}}\right)$$

$$= P\left(\frac{a}{10/\sqrt{35}} \leq Z\right) + P\left(Z \leq -\frac{a}{10/\sqrt{35}}\right)$$

$$= 2\left(1 - \Phi\left(\frac{a}{10/\sqrt{35}}\right)\right)$$

$$\Rightarrow \Phi\left(\frac{a}{10/\sqrt{35}}\right) = 0.975$$

$$\Rightarrow \frac{a}{10/\sqrt{35}} = 1.96 \Rightarrow a = 3.313.$$

So critical region has critical values & is given by

$50 - 3.313 = 46.687 \leftarrow \bar{x} \leq 46.687$

$50 + 3.313 = 53.313 \leftarrow \bar{x} \geq 53.313.$