3Y03 - 3J04 PROBABILITY & STATISTICS FOR (CIVIL) ENGINEERING

C01 - Lecture 2

STATISTICS: the science of collecting, describing, analysing (numerical) data & inferring (learning) information about the whole from some representative sample

PROBABILITY (THEORY): the mathematics of random phenomena.

PROBABILITY THEORY

The Language of Probability

Outcomes v. Events

SAMPLE SPACE : - set of all possible "outcomes" of an "experiment"

└ denoted by S

- can be discrete (finite or countably infinite)
  continuous (contains an interval)

Examples (i) Roll a regular 6-sided die

$S = \{1, 2, 3, 4, 5, 6\}$

(discrete)

(ii) Amount of money in envelope (possibly none)

$S = \{ x \mid x \geq 0 \}$

If you know there are at most \$50

then $S = \{ x \mid 0 \leq x \leq 50 \}$

continuous

(iii) Who's going to win the World Series? next year

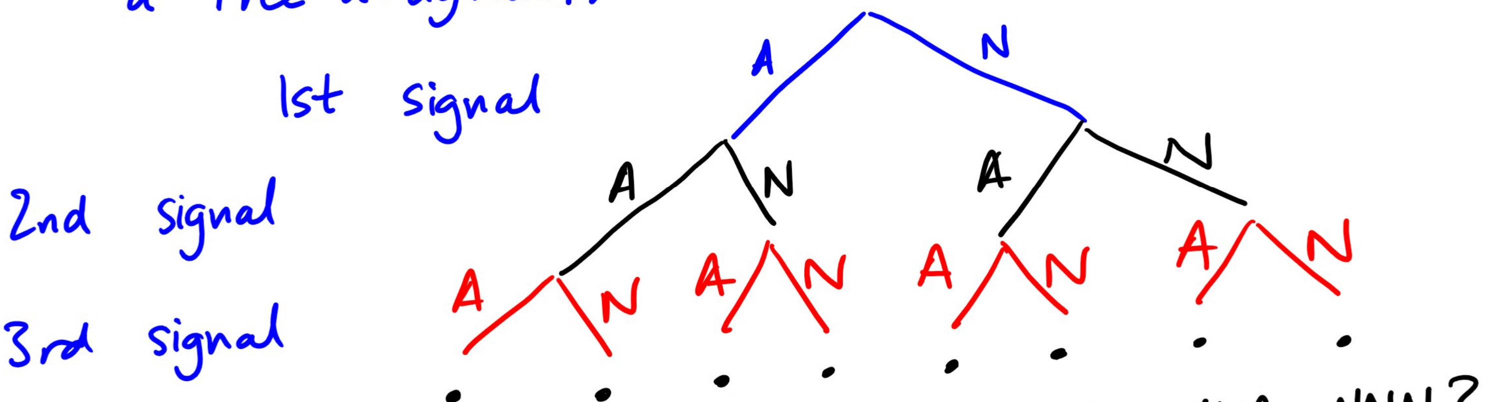
{ Jays, Red Sox, Yankees, ... } 30 teams

This year { Red Sox, Yankees, ... } 26 teams

The choice of sample space made at the outset of analysing a problem will change how we analyse it, and sometimes, different choices are possible.

(iv) Send 3 signals in succession using a communication system & record if they each arrive (A) or not (N)

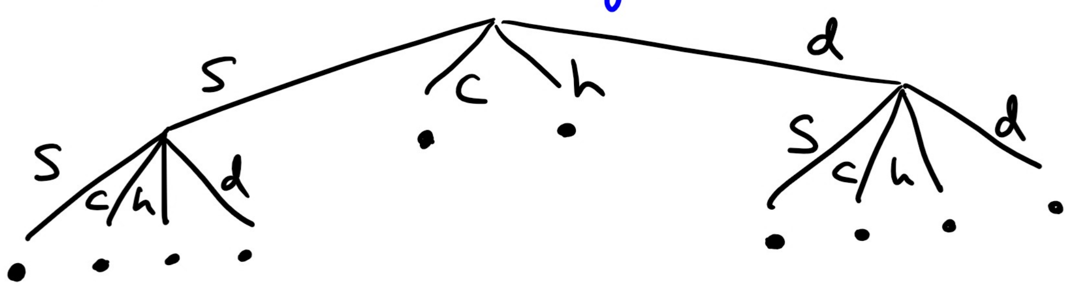
Set of possible outcomes can be found by using a tree diagram.



S = { AAA , AAN, ANA, ANN, NAA, NAN, NNA, NNN }

(v) Draw a card from a deck (52 cards). If clubs or hearts, stop. If spades or diamonds, draw another card.

Again draw a tree diagram:



S = { ss, sc, sh, sd, c, h, ds, dc, dh, dd }

Events Some subset/multiset of samplespace S.

(i) E = roll a 6 on a 6-sided die.

(ii) E = between \$20 and \$30 in the envelope i.e.

$$E = \{ x \mid 20 \le x \le 30 \}$$

(iii) E = A team from the AL wins the World Series

(iv) E = exactly 2 signals (from 3) arrive (A).

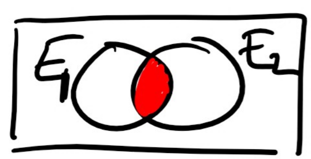
(v) E = we draw a club at some point.

E

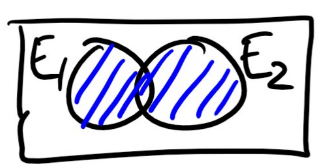
$$S = \{AAA, AAN, ANA, ANN, NAA, NAN, NNA, NNN\}$$

We need to work with sets and subsets so we need to know about

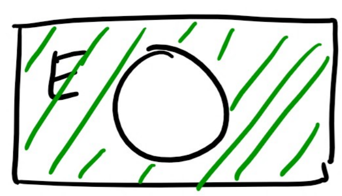
<u>Intersection</u> $E_1 \cap E_2$



<u>Union</u> $E_1 \cup E_2$

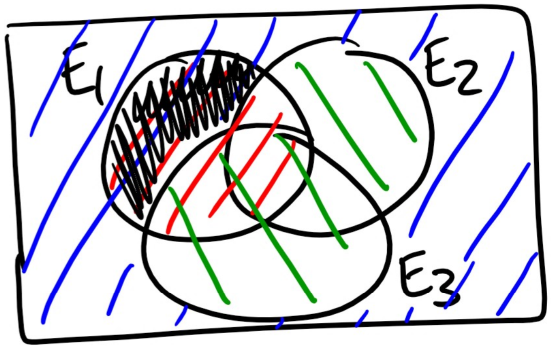


<u>Complement</u> E^c or E'



We say two events E_1 and E_2 are <u>mutually exclusive</u> if $E_1 \cap E_2 = \phi \leftarrow$ empty set

Example Illustrate $\underline{E_1} \cap \underbrace{(\underline{E_2 \cup E_3})^c}$ using a Venn Diagram.



Useful in this context:

Distributive Laws:

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$
$$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$$

De Morgan's Laws:

$$(A \cup B)^c = A^c \cap B^c$$
$$(A \cap B)^c = A^c \cup B^c.$$

Counting Techniques

Very often we want to know how many outcomes we have in our sample space or appear in a given event.

Earlier, with "3 signals" example ((iv)), tedious to write out all 8 outcomes.
(Worse with 100,000 signals!)
Instead we have tricks:

Multiplication Rule

Suppose each outcome is the result of a series of k steps, and there are

n_1 ways of completing step 1
n_2 " " " " 2 no matter what preceded
$\vdots$ $\vdots$
n_k " " " " k "

Then the total # possible outcomes is

$$n_1 \times n_2 \times \ldots \times n_k.$$

So for our "3 signals" problem, with 2 possibilities at each step, and 3 steps, we have $2 \times 2 \times 2 = 2^3 = 8$ possible outcomes.