3Y03 - 3J04 PROBABILITY & STATISTICS FOR (CIVIL) ENGINEERING

C01 - Lecture 3

Last time Sample Space S of outcomes
Subsets of S are events.

Multiplication Rule

Outcome after k steps; n_i ways of completing step i

Total # possible outcomes $= n_1 \times n_2 \times \ldots \times n_k$.

Example Licence plates have 4 letters followed by 3 single digit numbers.

(a) How many different licence plates are possible?

(b) What if repetition of entries is not allowed?

Solution (a) L L L L # # #

$$26 \times 26 \times 26 \times 26 \times 10 \times 10 \times 10 = 26^4 \times 10^3$$
$$= 456,976,000$$

(b) L L L L # # #

$$\underbrace{26 \times 25 \times 24 \times 23}_{= \frac{26!}{22!}} \times \underbrace{10 \times 9 \times 8}_{= \frac{10!}{7!}} = 258,336,000$$

Permutations

A *k-Permutation* of n distinct objects is an ordered arrangement of k of those objects.

How many such are there?

n choices for object #1

$n-1$ choices for object #2

$\vdots \qquad\qquad\qquad \vdots$

$(n-k+1)$ choices for object #k

So, by Mult. Rule, # k-Permutations from n objects $n \times (n-1) \times \ldots \times (n-k+1)$

$$= \boxed{\frac{n!}{(n-k)!} =: P^n_k} .$$

Permutation of Similar Objects

Suppose in our "3 signals" example ((iv)) we consider event "exactly 2 signals arrive (A)"

In how many ways can that happen?

It means getting 2 As and 1 N.

So our question is in how many ways can we arrange 2 As and 1 N?

Permutations of Similar Objects Rule

If we have n objects of r different types with n_1 of type 1, n_2 of type 2, $\dots$, n_r of type r, then $(n = n_1 + \dots + n_r)$ the number of (n-)permutations of our n objects is

$$\boxed{\frac{n!}{n_1! \, n_2! \cdot \dots \cdot n_r!}}$$

Why?

If all objects of different types, we just have $P^n_n = n!$

But $A_1 A_2 N$ should be indistinguishable from $A_2 A_1 N$

(We're arranging 2 As & 1 N in some order, & suppose we tag the two As to keep track.)

Idea: $n_1!$ ways of rearranging the objects of type 1 (while fixing the others) within any given string, so divide out by $n_1!$

Do the same for each type: end up dividing by $n_2!, \dots, n_r!$

So in the 3 signals question above, $n = 3$

$n_1 = 2$, $n_2 = 1$; so answer is $\dfrac{3!}{2!\,1!}$

$\nearrow$ # As $\quad\nearrow$ # Ns

$= 3$.

<u>Example</u> Suppose a hospital operating room has to schedule 2 hip surgeries, 3 knee surgeries & 1 shoulder surgery. → 6 surgeries

(a) How many poss. schedules are there, if all that matters is type of surgery?

(b) How many if knee surgery happens at start and the end?

KHHKSK
HSKHKK

<u>Solution</u> (a) $\dfrac{6!}{2!\,3!\,1!} = \cancel{420.}$ 60

(b) We fix K start & end, so the problem becomes: schedule in 4 slots the remaining 2 hip, 1 shoulder & 1 knee surgery i.e. $\dfrac{4!}{2!\,1!\,1!} = 12$.

<u>Combinations</u> With Permutations we had <u>ordered</u> arrangements. Now we only care about content, not order, in selection.

Example 20 potential jurors from which 12 are chosen. How many ways can this be done?

(If we cared about the order they sit, this is P^{20}_{12}.)

If order does not matter, then we use this rule (to eliminate from ordered setting repetitions with the same group of people).

r-Combinations

The # subsets of r elements that can be chosen from n objects is given by $\boxed{C^n_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}}$

"n choose r"

i.e. # juries of 12 from 20 is $C^{20}_{12} = \frac{20!}{12!\,8!}$

$= 125,970.$

Example Now suppose 15 of the 20 people are men & 5 are women.

How many juries of 12 are there with exactly 2 women?

<u>Solution</u> 1st choose 2 women from 5 i.e C^5_2

2nd " 10 men " 15 i.e. C^{15}_{10}

& use multiplication rule : $C^5_2 \times C^{15}_{10}$

(There are two steps, choosing women & choosing men, and the # ways of completing each of those steps is given above.)

$$= \frac{5!}{2!\,3!} \times \frac{15!}{10!\,5!}$$

$$= 30{,}030.$$

<u>Example</u> Batch of 100 semiconductor chips

75 are conforming

25 are not " .

Select a sample of 3 chips to test.

(a) # different samples? $= \binom{100}{3}$

(b) # " " with exactly one non-conforming chip?

$$= \binom{25}{1} \times \binom{75}{2}.$$

(c) # different samples with at least 1 non-conf. chip?

2 ways: $\binom{25}{1}\times\binom{75}{2} + \binom{25}{2}\times\binom{75}{1}$ ← exactly 2

exactly 1 non-conforming

$+ \binom{25}{3}\times\binom{75}{0}$ ← exactly 3

# of ways all conform : $\binom{75}{3} \times \binom{25}{0}$

So answer $= \binom{100}{3} - \binom{75}{3} \times \binom{25}{0}$.

(Check you get same answer each way!!)