3Y03 - 3J04 PROBABILITY & STATISTICS FOR (CIVIL) ENGINEERING

C01 - Lecture 4

PROBABILITY - for now only work with discrete sample spaces

- Subjective : best guess e.g. betting odds
  - elections, football games
  - non-repeatable experiments
- Objective : relative frequency of events
  - repeating experiments

  "Things settle down in the long run"

Probability : - function assigned to events of sample space
- represents belief in what the outcome of an experiment will be

Axiomatic approach : mathematical tools & rules to understand chance

Suppose S finite e.g. $S = \{s_1, \dots, s_N\}$ ← N v. large typically

Assign weights to s_i : a non-negative # p_i assigned to s_i

where $\sum_{i=1}^{N} p_i = 1$. ← so notice $0 \le p_i \le 1$ for each i.

For an event $E \ (\subseteq S)$, the probability of E

$$P(E) = \sum_{\substack{i \\ \text{where } s_i \in E}} p_i \quad .$$

Example $S = \{a, b, c\}$, with weights $\underbrace{\{0.1, 0.5, 0.4\}}_{\text{add up to } 1}$

If $A = \{a, c\}$, $B = \{b, c\}$, what is

(a) $P(A)$, (b) $P(B)$ (c) $P(A \cap B)$?

Solution (a) $P(A) = 0.1 + 0.4 = 0.5$

(b) $P(B) = 0.5 + 0.4 = 0.9$

(c) $P(A \cap B) = P(\{c\}) = 0.4$.

Simplest situation is that all outcomes are equally likely i.e. everything is random i.e.

$[S = \{s_1, \dots, s_N\}]$ all p_is same? $\longrightarrow$ i.e. $p_i = \frac{1}{N}$ for all i.

So for any event $E \ (\subseteq S)$ $P(E) = \frac{|E|}{N}$ ← size of E.

Example 100 chips, 75 conforming
25 non-conforming

3 sampled randomly without replacement.

E = getting exactly 2 non-conforming chips.

Q: P(E)?

Solution $P(E) = \frac{|E|}{N} = \frac{\binom{25}{2} \times \binom{75}{1}}{\binom{100}{3}} = \frac{300 \times 75}{161{,}700} \approx 0.14.$

If we introduce a variable X for # non-conf. chips then E is the event $X = 2$ ← X is a **random variable**

$[P(X=2)]$

A **probability distribution** tells us the probability associated with each value of X:

e.g. here:

X	$P(X)$
0	0.42
1	0.43
2	0.14
3	0.01
	1

← should add up to 1:

Axioms of Probability – rules; allow us to draw inference about probabilities based on observed/'known' information.

(1) $P(S) = 1$ – prob. of something happening is certain

(2) $0 \leq P(E) \leq 1$

(3) If $E_1 \cap E_2 = \phi$, ← E_1, E_2 mutually exclusive
then $P(E_1 \cup E_2) = P(E_1) + P(E_2)$.

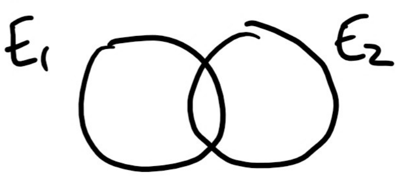
Notice $P(E^c) = 1 - P(E)$.
(in particular $P(\phi) = 0$.)

Can extend (3) to as many events as you like i.e. $E_1, \dots, E_k$ and $E_i \cap E_j = \phi$ for each pair $i \neq j$

has $P(E_1 \cup \dots \cup E_k) = P(E_1) + \dots + P(E_k)$.

<u>Question</u> What if $E_1, \dots, E_k$ are not (pairwise) mutually exclusive? What is $P(E_1 \cup \dots \cup E_k)$?

<u>Addition Rule</u> $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$



Think area!

<u>Example</u>

	Passed	Failed	TOTAL
CO1	123	48	171
CO2	115	32	147
TOTAL	238	80	318

Person selected at random

E_1 : in CO2 $\qquad E_2$: passed

(a) $P(E_1) = \dfrac{147}{318}$. (b) $P(E_2) = \dfrac{238}{318}$

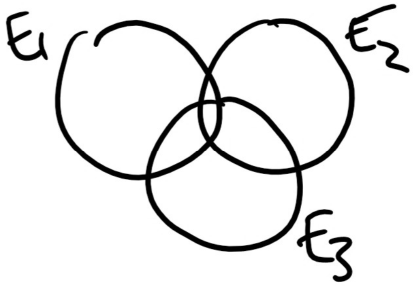
(c) $P(E_1 \cap E_2) = \dfrac{115}{318}$

(d) $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$$= \frac{147 + 238 - 115}{318} = \frac{270}{318}.$$

Addition Rule for 3 events

$$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3)$$



$$- P(E_1 \cap E_2) - P(E_1 \cap E_3) - P(E_2 \cap E_3)$$

$$+ P(E_1 \cap E_2 \cap E_3)$$

Example Passwords: 8 characters; upper case letters (26) lower case letters (26); digits (10)

What is the probability of having $\underbrace{\text{\#2 at start}}_{E_1}$ or $\underbrace{\text{K in the 5th spot}}_{E_2}$?

Solution

$$P(E_1 \overset{\text{OR}}{\cup} E_2) = P(E_1) + P(E_2) - \overset{\text{BOTH}}{P(E_1 \cap E_2)}$$

$$= \frac{62^7}{62^8} + \frac{62^7}{62^8} - \frac{62^6}{62^8}$$