3Y03 - 3J04 PROBABILITY & STATISTICS FOR (CIVIL) ENGINEERING

C01 - Lecture 5

Last time - $P(E)$ defined for events E of S

AXIOMS: • $P(S) = 1$; • $0 \le P(E) \le 1$

• $P(E_1 \cup E_2) = P(E_1) + P(E_2)$ if $E_1 \cap E_2 = \phi$

i.e. E_1, E_2 mutually exclusive.

[same if $P(E_1 \cap E_2) = 0$ i.e. !!!]

Addition Rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Conditional Probability

- Update idea of how likely an event is based on new information

$P(A \mid B)$ is the conditional probability of "A given B"

Example Failure of heating systems:

	E	E^c	TOTAL
G	32	5	37
G^c	21	12	33
TOTAL	53	17	70

E : electrical failure

G : gasleak

What is $P(G)$? $\quad P(G) = \frac{37}{70}$.

? $P(G|E) = \frac{32}{53}$ $\quad$? $P(G \cap E) = \frac{32}{70}$.

$P(E|G) = 32/37$

$$\boxed{\text{FORMULA FOR CONDITIONAL PROBABILITY: } \quad P(A|B) = \frac{P(A \cap B)}{P(B)}}$$

(when everything equally likely

$$= \frac{\#\text{outcomes in B among those in A}}{\#\text{outcomes in B}}$$

We can rewrite this as

$$\boxed{\begin{aligned} P(A \cap B) &= P(A|B)P(B) \\ \text{also} \quad P(A \cap B) &= P(B|A)P(A) \end{aligned}}$$ Choose which to use depending on the situation.

MULTIPLICATION RULE (for conditional probability)

<u>Example</u> 200 items, 10 defective, take sample of size 2 without replacement.

(a) $P(\text{second item chosen defective} \mid \text{first one is defective})$

$E_2 = \{$2nd defective$\}$ $\qquad$ $E_1 = \{$first defective$\}$

$$P(E_2 | E_1) = 9/199$$ (after picking 1 defective, 9 defectives left in 199)

(b) $P(E_1 \cap E_2) = P(E_2 | E_1) . P(E_1)$

$$= 9/199 \times 10/200$$

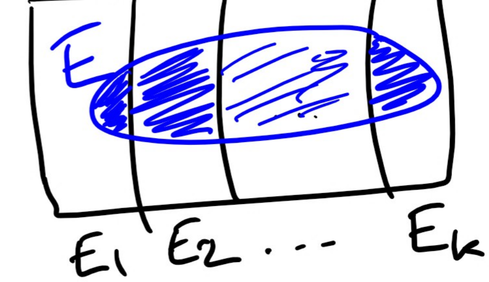
Conditional prob. formula v. useful in situations where problem is set up as a chain of steps.

Total Probability

You can recover the probability $P(E)$ of event E from knowing: prob. of E under each of an exhaustive list of conditions i.e. a partition of S

Divide up S into events $E_1, \dots, E_k$ that are pairwise mutually exclusive

$$E_i \cap E_j = \phi, \; i \neq j$$



$(S = E_1 \cup \dots \cup E_k)$

$$P(E) = P(E \cap E_1) + P(E \cap E_2) + \dots + P(E \cap E_k)$$

$$P(E) = P(E|E_1)P(E_1) + P(E|E_2)P(E_2) + \dots + P(E|E_k)P(E_k)$$

using multiplication formula.

TOTAL PROBABILITY RULE

Example Manufacture semi-conductors

You know the prob. that they fail depending on level of contamination : Low, Medium, High

i.e. we know

	$P(\text{Fail} \mid \text{Level})$	Level
$P(\text{Fail} \mid H) =$	0.2	H
$P(\text{Fail} \mid M)$	0.05	M
$P(\text{Fail} \mid L)$	0.0001	L

You also know $P(H) = 0.1$, $P(M) = 0.3$, $P(L) = 0.6$

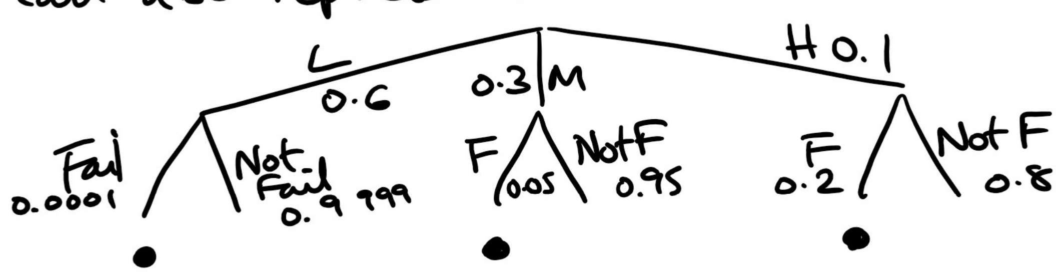
Find $P(\text{Fail})$.

Solution
$$P(\text{Fail}) = P(\text{Fail} \mid H) \cdot P(H) + P(\text{Fail} \mid M) P(M) + P(\text{Fail} \mid L) P(L)$$

$$= 0.2 \times 0.1 + 0.05 \times 0.3 + 0.0001 \times 0.6$$

$$= \underline{\underline{0.03506}}.$$

We can also represent this as tree diagram:



← Partition of S

← Conditional prob. given first step

P(Fail) = sum of what you get when you multiply along all the branches with Fail at bottom (• above).

<u>Example</u> Schedule surgeries : 2 knee KK
3 hip HHH
2 shoulder SS

All schedules equally likely!

P(all knee surgeries last | all shoulder first)

$$= \frac{P(\text{ all knee last AND all shoulder first})}{P(\text{all shoulder first})}$$

SS HHHKK

everything else

SS _ _ _ _ _

$$= \frac{2!3!2!}{7!} \Big/ \frac{2!5!}{7!} = \frac{\cancel{2}!3!2!}{\cancel{2}!5!} = 1/\binom{5}{2}.$$

If we assume all shoulders, knee, hip identical (within categories) then:

SS | _ _ _ _ _ choose where to put 2×K in here [$\binom{5}{2}$ possibilities]

& only 1 of those has _ _ _ KK

→ so prob. = $1/\binom{5}{2}$.