3Y03 - 3J04 PROBABILITY & STATISTICS FOR (CIVIL) ENGINEERING

C01 - Lecture 6

Last time CONDITIONAL PROBABILITY

MULTIPLICATION RULE $P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)$

TOTAL PROBABILITY if $S = E_1 \cup \ldots \cup E_k$ and $E_1, \ldots, E_k$ pairwise mutually exclusive

$$P(E) = P(E|E_1)P(E_1) + \ldots + P(E|E_k)P(E_k)$$

Independence If A has no effect on the prob. of B i.e. $P(B|A) = P(B)$

then events A and B are independent.

(So also $P(A|B) = P(A)$.)

Notice (from conditional prob formula) that

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = P(B)$$

↑ if A, B independent

i.e.

If A, B independent, then $P(A \cap B) = P(A)P(B)$.

We can extend this to any # of events:

$E_1, \dots, E_k$ are <u>independent</u> if

$$P(E_{i_1} \cap \dots \cap E_{i_j}) = P(E_{i_1}) \times P(E_{i_2}) \times \dots \times P(E_{i_j})$$

for any subcollection $E_{i_1}, \dots, E_{i_j}$ from $E_1, \dots, E_k$.

Independence modelled based on knowledge about experiment ie maybe allowed to assume independence based on situation.

<u>Example</u> Sample with replacement.

500 parts, 493 non-defective, 7 defective.
6 samples <u>with</u> replacement.

E_1 : 1st one defective
E_2 : 2nd one defective

$$P(E_2 \mid E_1) = P(E_2) \quad \text{because of replacement}$$
$$= 7/500.$$

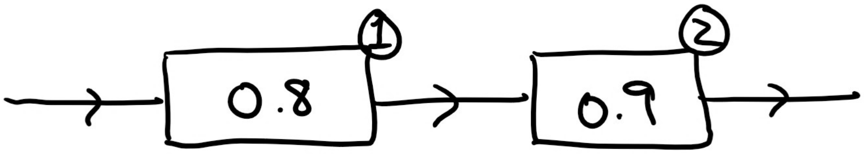
$$P(E_1 \cap E_2) = P(E_1)\,P(E_2) = \frac{7}{500} \times \frac{7}{500} = \frac{49}{250000}$$

<u>Observation</u> If A, B independent then so are the pairs A^c, B^c ; A^c, B ; A, B^c.

Circuits & Reliability

→ Probability that a device/system operates e.g. for some required duration

Example



SERIES CIRCUIT

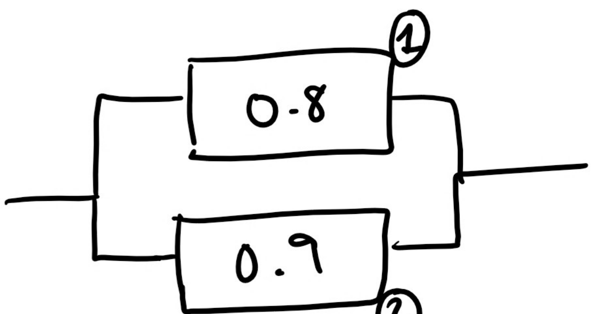
E_1 : device 1 functions

E_2 : " 2 "

Devices fail independently.

$$P(\text{circuit operates}) = P(\text{both devices operate})$$
$$= P(E_1 \cap E_2)$$
$$= P(E_1)\,P(E_2) = 0.8 \times 0.9$$
$$= \underline{\underline{0.72}}$$

Example



(Again operation/failure of devices is independent)

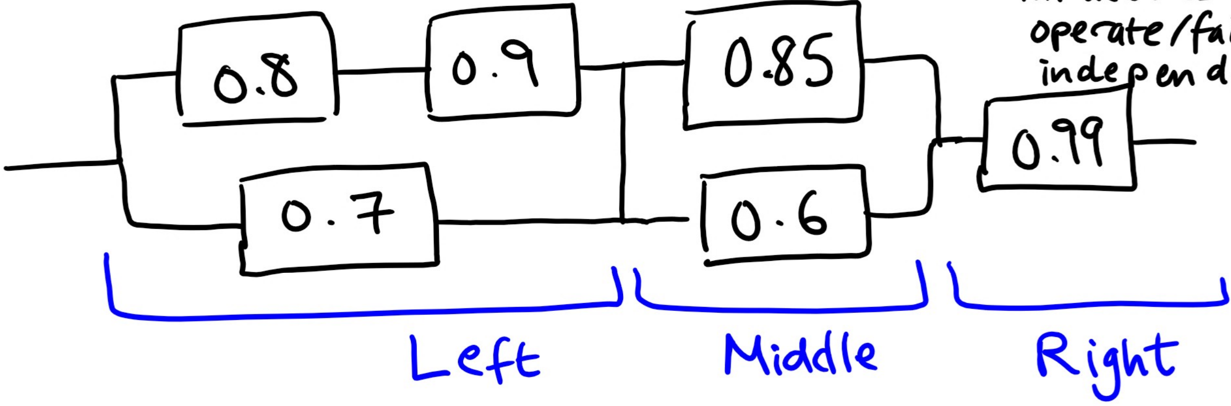
$$P(\text{circuit operates}) = P(E_1 \cup E_2)$$
$$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

PARALLEL CIRCUIT

$$= 0.8 + 0.9 - P(E_1)P(E_2)$$
$$= 0.8 + 0.9 - (0.8)(0.9)$$
$$= \underline{\underline{0.98}}$$

Example

All devices operate/fail independently



MIXED CIRCUIT

↑ Break up into series and parallel circuits

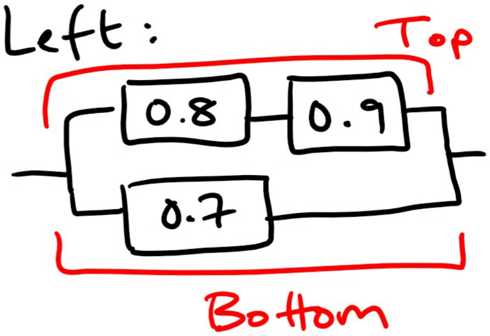
P(device operates)

= P(L operates ∩ M operates ∩ R operates)

= P(L operates) P(M operates) P(R operates)

(by independence)

Left:



P(L operates)

= P(T operates ∪ B operates)

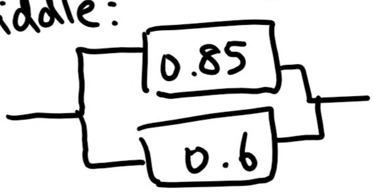
= P(T operates) + P(B operates) − P(both T and B operate)

$$= \overbrace{0.8 \times 0.9}^{\text{(series)}} + 0.7 - P(\text{T operates})P(\text{B operates})$$

$$= 0.8 \times 0.9 + 0.7 - ((0.8)(0.9)) \times 0.7$$

$$= \underline{\underline{0.916}}$$

Middle:



$$P(\text{M operates}) = \overbrace{0.85 + 0.6 - (0.85)(0.6)}^{\text{(parallel)}}$$

$$= \underline{\underline{0.94}}$$

So $P(\text{device operates}) = P(L)P(M)P(R) = (0.916)(0.94)(0.99) = \underline{\underline{0.85}}$.

Random Variables

↳ Represent possible numerical outcomes of an experiment

→ denoted by capital letters e.g. X

Just as with sample spaces, we have

Discrete random variables (range of r.v. is a finite set or countably infinite set)

Continuous random variables (range contains an interval, finite or infinite)

Example Chips analysed & # of surface flaws counted.

$X = \#$ flaws ← discrete random variable

$(X = 0, 1, 2, 3, 4, 5, \ldots)$

Example Play video game 5 times, independent games, stop after losing.

$P(\text{winning a game}) = 0.7$

$[P(\text{losing}) = 0.3]$ $X = \#$ wins, can take values in $\{0, 1, 2, 3, 4, 5\}$

Find $P(X=0), P(X=1), \ldots, P(X=5)$.