3Y03 – Probability and Statistics for Engineering

WS19 Lecture 11

Yesterday **Geometric/Negative Binomial Distribution**

→ X = # trials until 1st/rth success

$\mu = E(X) = \frac{r}{p}$, $\sigma^2 = V(X) = \frac{r(1-p)}{p^2}$ (p = probability of success in a single trial)

Hypergeometric Distribution → sample WITHOUT replacement

N: # items total n: # items sampled X = # 'successes' in sample

K: # 'success' items ($\leq N$) $\mu = E(X) = np$, $\sigma^2 = V(X) = np(1-p)\left(\frac{N-n}{N-1}\right)$

where p = P(choosing 'success' item from initial pool) $= \frac{K}{N}$.

Poisson Distribution – discrete

Count "incidencents" taking place randomly during period of time T

or "flaws" distributed randomly in a region of size T

e.g. # earthquakes in a period of 5 yrs

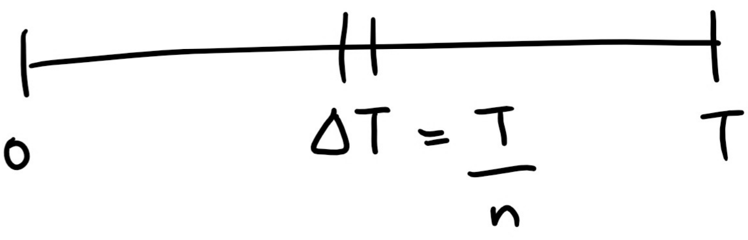
flaws in a pipeline in 10 km

defects in a plate of area 50 cm^2

To find p.m.f. : $X = \#$ incidents/flaws where we know <u>average</u> # per unit time/area

---

Brief Theoretical Digression :



Divide up length into pieces of length $\Delta T = \frac{T}{n}$ where prob. of > 1 incident/flaw in small piece size ΔT is negligible

(So ΔT very small i.e. n is very big.)

<u>Prob.</u> of incident/flaw in one ΔT piece is $\lambda \Delta T$

λ = average incidence rate per unit time/area

$$\left.\lambda \Delta T = \frac{\lambda T}{n}\right\} = p$$

& there are n intervals (of length ΔT)

So : $X \sim \text{Bin}(n, p)$

Ideally we have LOTS of little ΔT interval

So we take limit as $n \longrightarrow \infty$

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x} = \frac{n!}{x!\,(n-x)!} \left(\frac{\lambda T}{n}\right)^x \left(1 - \frac{\lambda T}{n}\right)^{n-x}$$

...

Let $n \longrightarrow \infty$ $\quad \boxed{\dfrac{(\lambda T)^x}{x!} e^{-\lambda T}}$

pmf of Poisson r.v. with parameter λ on region size T

So we can approximate Poisson distr. by Binomial by choosing some n.

X - Poission, λ, T:

$$E(X) = \mu = np \text{ as } n \to \infty$$

$$= n \frac{\lambda T}{n} = \lambda T$$

$$V(X) = \sigma^2 = np(1-p) \text{ as } n \to \infty$$

$$= \cancel{n} \frac{\lambda T}{\cancel{n}} \left(1 - \underbrace{\frac{\lambda T}{n}}_{\to 0}\right) = \lambda T$$

CAREFUL

Sometimes e.g. on formula sheet "λ" means "λT"

We say " X is a Poisson r.v. with mean _____ "

Example Disc surface contaminated with average of $\lambda = 0.3$ particles per cm^2 of surface

Discs have surface area $30\ cm^2$.
$\hookrightarrow T$ = Total area.

Find μ, $P(X=10)$, $P(X \leq 2)$.

$X = \#$ particles on one disc.

Solution $\mu = \lambda T = 0.3 \times 30 = 9$

$$P(X = 10) = \frac{(\lambda T)^x}{x!} e^{-\lambda T} = \frac{9^{10}}{10!} e^{-9}$$

x

$$= 0.1186$$

$$P(X \leq 2) = \sum_{x=0}^{2} \frac{9^x}{x!} e^{-9} = e^{-9}\left(1 + 9 + \frac{9^2}{2}\right)$$

$$= 0.0062.$$

Example Calls come into an exchange at rate of 5/hour. Let $X = \#$ calls in 3 hour period.

$\lambda = 5$ $\quad = T$

Find μ, $P(X=7)$.

Solution $E(X) = \mu = \lambda . T = 5 \times 3 = 15.$

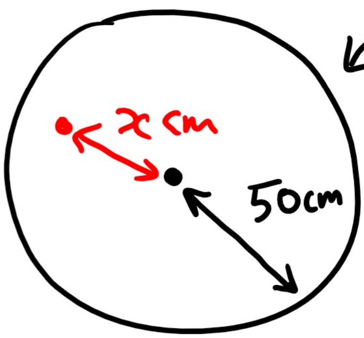
$$P(X = 7) = f(7) = \frac{(\lambda T)^7}{7!} e^{-\lambda T} = \frac{15^7}{7!} e^{-15} = 0.01.$$

Chapter 4 CONTINUOUS RANDOM VARIABLES.

X = some measurement/quantity taking values in an interval (maybe all real intervals)

Examples
- Time until failure of a machine
- Exact length of a manufactured part
- Electric current in a wire
- Measurement error in an instrument
- Net weight per packet

Example



Dartboard

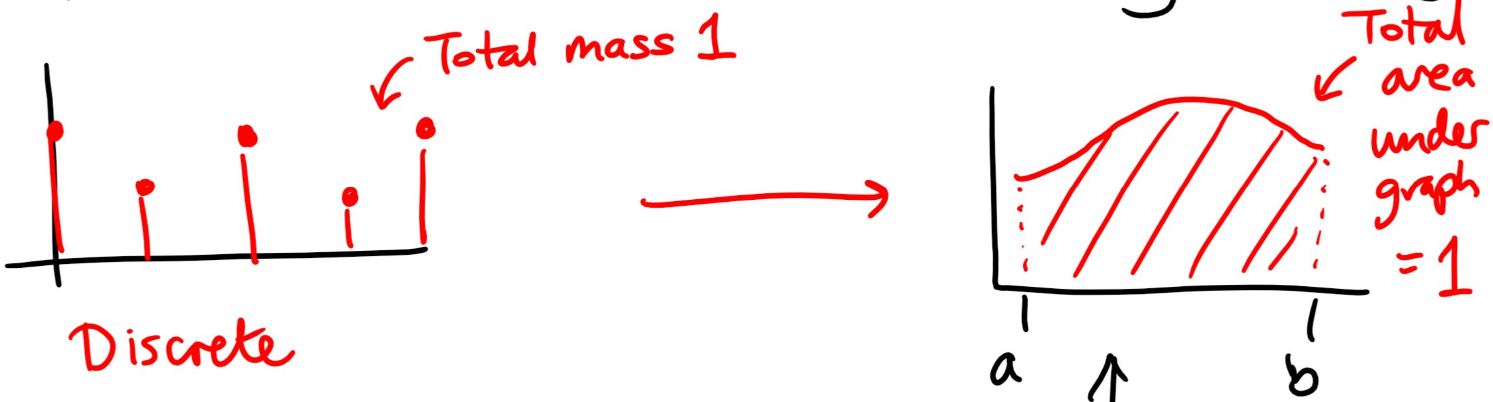
Distance of dart from centre = x cm.

X = distance of dart from centre

X can take values in $[0, 50]$

Notice: ∞ many possible values so
" $P(X = x) = \frac{1}{\infty} = 0$ " for fixed x

Instead we talk about Probability Density



$$\text{Total area} = \int_a^b f(x)\,dx$$

<u>Probability Density Function for X</u>.