3Y03 - Probability and Statistics for Engineering

WS19 Lecture 13

Last time

X - continuous random variable

$f(x)$ - probability density function [bounded]

$$\int_{\text{range of } X} f(x)\,dx = 1 \quad \text{and} \quad P(\underbrace{a \overset{\text{or } <}{\le} X \overset{\text{or } <}{\le} b}) = \int_a^b f(x)\,dx$$

also works for infinite intervals so we get:

cumulative distribution function: $F(x) = P(X \le x) = \int_{\text{bottom of range}}^{x} f(t)\,dt$

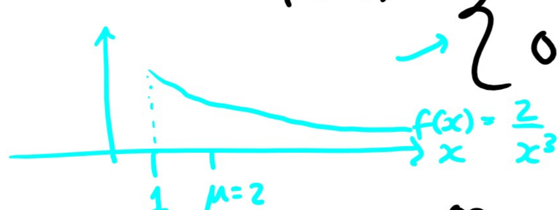
For any $h(X)$, $E(h(X)) = \int h(x) f(x)\,dx$ and $f(x) = F'(x)$.

e.g. $\mu = E(X) = \int x f(x)\,dx$, $\sigma^2 = V(X) = \int (x-\mu)^2 f(x)\,dx$

$$= E(X^2) - \left[E(X)\right]^2$$

Example The size of particles of contamination is modelled by p.d.f. $f(x) = \begin{cases} 2/x^3, & x > 1 \\ 0 & \text{o/w.} \end{cases}$

Find μ, σ^2, σ.



$$\mu = \int_1^\infty x\left(\frac{2}{x^3}\right)dx = \int_1^\infty \frac{2}{x^2}\,dx = \left[-2x^{-1}\right]_1^\infty = 2$$

$$\sigma^2 = E(X^2) - (E(X))^2 = \int_1^\infty x^2\left(\frac{2}{x^3}\right)dx - (2)^2$$

$$= \left[2\ln x\right]_1^\infty - 4 = \infty$$

"$\sigma = \infty$"

4.6 Normal Distribution AKA Gaussian Distribution

→ bell-shaped curve

↳ experiment repeated many times
+ average/total over all repetitions
Very often this ↑ has Normal distribution.

It has a strange p.d.f.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \quad \text{for all } x.$$

There are parameters μ, σ with $\mu \in \mathbb{R}$, $\sigma > 0$.

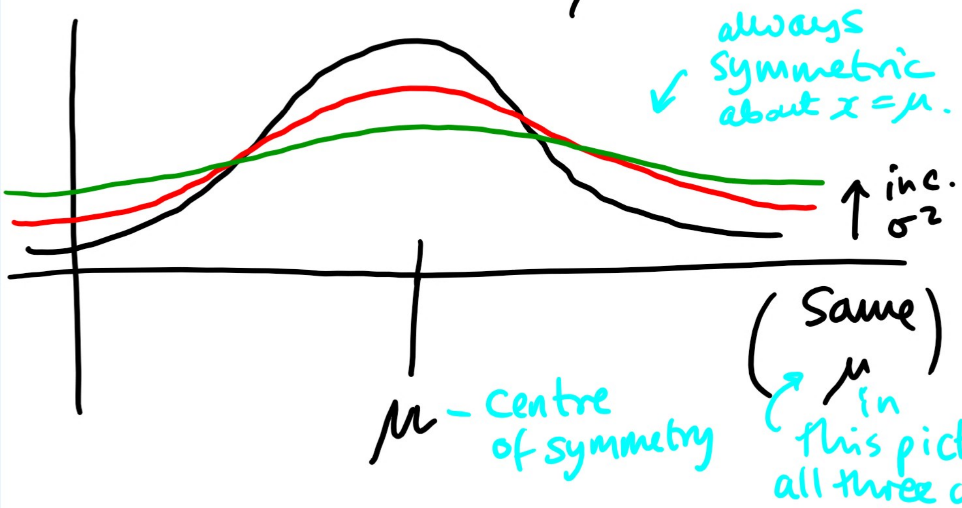
These are suggestively named:
If X has normal distribution with parameters μ and σ^2 (written $X \sim N(\mu, \sigma^2)$),
then $E(X) = \mu$ $V(X) = \sigma^2$.

Working out $F(x) = P(X < x)$ i.e. integrating

$f(x)$ not realistically possible (no closed form expression for $F(x)$).

So instead we use values from a table.

For each pair (μ, σ^2) we have a p.d.f. $f(x)$:



The higher the value of σ^2, the flatter the curve

(Different (μ, σ^2) give different p.d.f. $f(x)$.)

There is a special one called a <u>standard Normal distribution</u> $Z \sim N(0, 1)$ with

c.d.f. $\Phi(z) = P(Z \leq z)$.

Impractical to have tables for integral values $(F(x))$ for every pair (μ, σ^2); we have <u>one table</u>, the one for $\Phi(z)$, i.e. for $N(0, 1)$.

Recipe: Standardize any other normal r.v.:

If $X \sim N(\mu, \sigma^2)$, then $Z = \dfrac{X-\mu}{\sigma} \sim N(0,1)$

So $P(\boxed{X \le x}) = P\left(\boxed{\dfrac{X-\mu}{\sigma} \le \dfrac{x-\mu}{\sigma}}\right)$

This inequality is true exactly when this other inequality is true, so they have equal prob. of being true.

$= P(Z \le z)$ ← just rename things!

↖ where $z = \dfrac{x-\mu}{\sigma}$

$= \Phi(z)$ ← This is the value in the table.

Example File transfer from server to computer is normally distributed with mean 5.75 Mbps and variance 0.35^2 Mbps^2.

(a) Find prob. speed is 6.7 Mpbs or more.

(b) Find prob. speed is 5.5 Mbps or less.

Solution X = transfer speed $\sim N(5.75, 0.35^2)$

(a) $P(X \geq 6.7) = 1 - P(X < 6.7)$

$= 1 - P\left(Z < \dfrac{6.7 - 5.75}{0.35}\right)$

$= 1 - P(Z < 2.71)$

$= 1 - 0.996636 = 0.003364.$

(b) $P(X \leqslant 5.5) = P\left(Z \leqslant \frac{5.5 - 5.75}{0.35}\right)$

$= P(Z \leqslant -0.71)$

$= 0.238852.$