3Y03 – Probability and Statistics for Engineering

WS19 **Lecture 2**

Yesterday

Experiment: anything that produces data

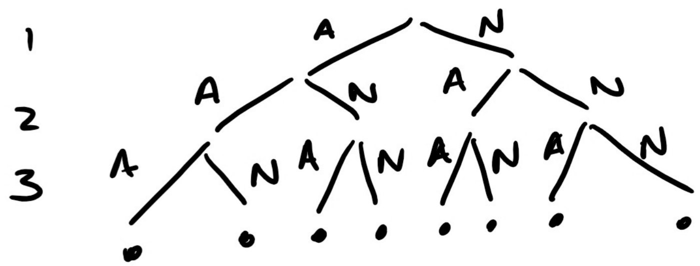
Statistics: science of data: collecting, describing, analysing, inferring info.

Probability: mathematics of randomness

SAMPLE SPACE S = set of all possible **outcomes** of an experiment

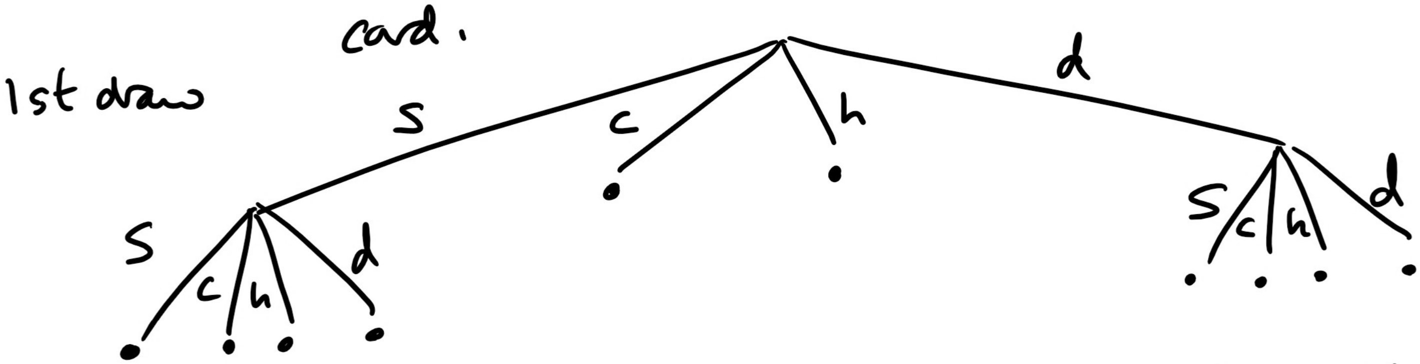
discrete (can count it) / **continuous** (interval)

Reminder 3 signals: arrive (A) or don't (N)



$S = \{AAA, AAN, ANA, \ldots, NNA, NNN\}$

Example: Draw a card from deck. If clubs or hearts stop. If spades or diamonds draw another card.



$S = \{ss, sc, sh, sd, c, h, ds, dc, dh, dd\}$ (discrete)

Events

Some subset/multiset of the sample space.

Notice how all the "Events" are given in terms of the content of the corresponding sample space (above or last lecture)

e.g. (i) $E =$ roll a 6 on a 6-sided die

$= \{6\}$ Recall $S = \{1, 2, 3, 4, 5, 6\}$

(ii) $E =$ between \$20 and \$30 in envelope

$= \{x \mid 20 \leq x \leq 30\}$ Recall $S = \{x \mid x \geq 0\}$

(iii) $E =$ A team from the National Conference wins the Superbowl $= \{\ \ ,\ \ ,\ \ ,\ \ \}$

4 teams

Recall: $S = \{$8 teams remaining in NFL competition$\}$

(iv) $E =$ exactly 2 signals from 3 arrive

$= \{AAN, ANA, NAA\}$ Recall: $S = \{AAA, AAN, ANA, ANN, NAA, NAN, NNA, NNN\}$.

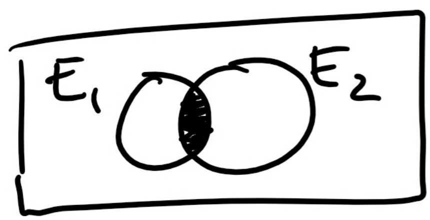
(v) $E =$ we draw a club at some point

$= \{sc, dc, c\}$ Recall: $S = \{ss, sc, sh, sd, c, h, ds, dc, dh, dd\}$

So we're working with sets & subsets so the following will be useful:

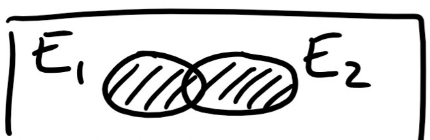
Intersection $E_1 \cap E_2$

In E_1 AND E_2



Union $E_1 \cup E_2$

In E_1 OR E_2 (poss. both)



Complement $E^c = E'$ NOT

We say that two events E_1, E_2 are <u>mutually exclusive</u> if $E_1 \cap E_2 = \emptyset$ ← the empty set

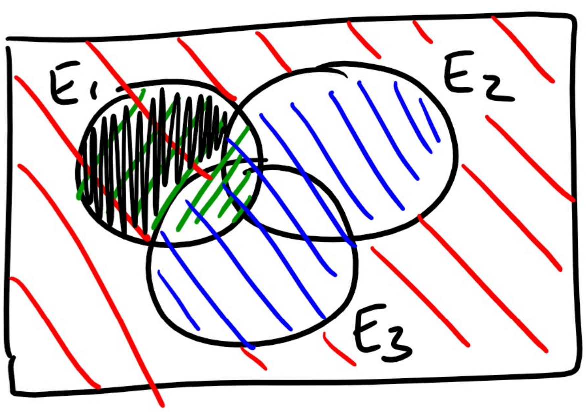
e.g. Roll a 6-sided die <u>once</u>

$$\left.\begin{aligned} E_1 &= \{4\} \text{ i.e. roll a } 4 \\ E_2 &= \{6\} \text{ i.e. roll a } 6 \end{aligned}\right\} \text{mutually exclusive}$$

i.e. mutually exclusive = no outcome in both events

<u>Example</u> Illustrate $\underline{E_1} \cap \overbrace{(\underline{E_2 \cup E_3})}^{}{}^c$ with a Venn diagram.

<u>Solution</u>



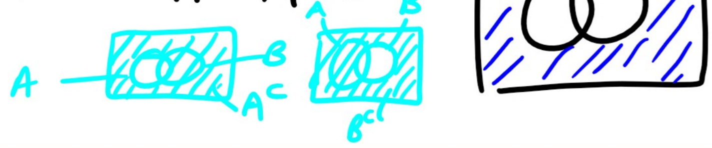
Useful in this context:

<u>Distributive Laws</u>: $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

$$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$$

<u>De Morgan's Laws</u>: $(A \cap B)^c = A^c \cup B^c$

$$(A \cup B)^c = A^c \cap B^c$$



Counting Techniques

Very often we need to know how many outcomes we have in the sample space or how many appear in a certain event.

Earlier: 3 signals → tedious to write out sample space / tree diagram

→ but told us there were 8 outcomes

But totally impractical tool for, say, 100000 signals.

So instead we have tricks to simplify matters:

Multiplication Rule

Suppose each outcome is the result of a series of k steps and there are:

n_1 ways of completing step 1

n_2 ways of completing step 2 no matter what preceeded

⋮

n_k ways of completing step k no matter what preceeded

Then the total # possible outcomes is $n_1 \times n_2 \times \ldots \times n_k$.

So for 3 signals, 2 possibilities (A or N) for each signal, so $n_1 = n_2 = n_3 = 2$ so

$$\# \text{ outcomes} = 2 \times 2 \times 2 = 2^3 = 8$$

<u>Example</u> Licence plates have 4 letters followed by 3 digits (#s)

(a) How many licence plates are possible?

(b) What if repetition of entries is not allowed?

<u>Solution</u> (a) $\underline{L}\ \underline{L}\ \underline{L}\ \underline{L} \,/\, \underline{\#}\ \underline{\#}\ \underline{\#}$

$$26 \times 26 \times 26 \times 26 \times 10 \times 10 \times 10 = 456,976,000$$
$$= 26^4 \times 10^3$$

(b) $L\ L\ L\ L / \#\ \#\ \#$

$$\underbrace{26 \times 25 \times 24 \times 23}_{= \frac{26!}{22!} = \frac{26!}{(26-4)!}} \times \underbrace{10 \times 9 \times 8}_{= \frac{10!}{7!} = \frac{10!}{(10-3)!}} = 258,336,000$$

$\frac{10!}{(10-3)!}$ ← # ways of choosing 3 #s without repetition

$\frac{26!}{(26-4)!}$ ← # ways of choosing 4 letters without repetition