3Y03 – PROBABILITY AND STATISTICS FOR ENGINEERING

WS19 Lecture 24

Today (& beyond) HYPOTHESIS TESTING

9.1 (General Setup)

Statistical Hypothesis: statement about parameters

e.g. $\mu = 50$, $\sigma_1^2 > \sigma_2^2$

Setup

initial claim or assumption → H_0 (null hypothesis) → equality statement

e.g. $\mu = 50$

$\sigma_1^2 = \sigma_2^2$

H_1 (alternative hypothesis) → statement contradicting H_0 (3 poss.)

e.g. $H_0: \mu = 50$, $\begin{cases} H_1: \mu \neq 50 \text{ (2-sided)} \\ \text{or} \\ H_1: \mu > 50 \\ \text{or } H_1: \mu < 50 \end{cases}$ 1-sided

Assume H_0. Look at data. Do you see something implausible i.e. very unlikely to happen if H_0 were true?

If yes, then reject H_0 (in favour of H_1).

If no, then accept H_0.

Objectives 1. Test if parameter value has changed
e.g. change production process
(H_0: old parameter value)

2. Test a theory (H_0: theoretical parameter value)

3. Conformance testing (H_0: standard expected for the parameter value)

Basic Idea ① Assume H_0 & look at how likely it is that "test statistic" takes given value ↳ $\bar{x}$, S^2, $\hat{p}$

First assume H_0; then pick a test statistic.

② Pick a test statistic

e.g. in testing

$$H_0 : \mu = 50$$
$$H_1 : \mu \neq 50$$

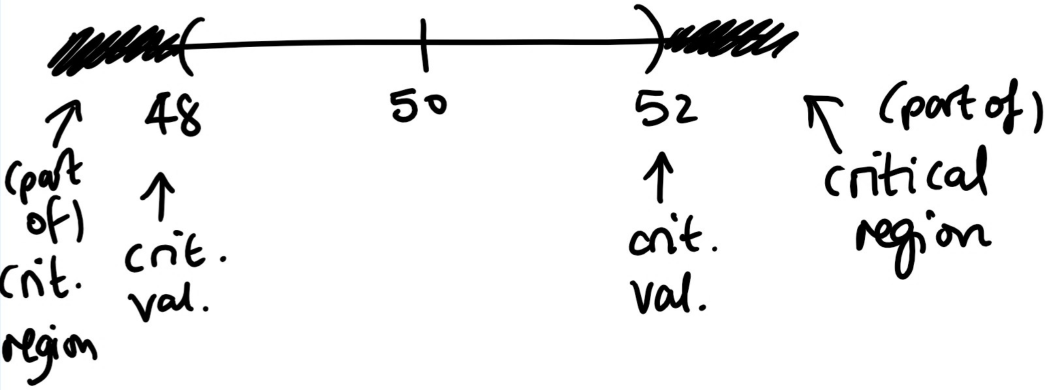
we'd use $\bar{X}$ as test statistic

③ Specify a critical region (by giving critical values)

↳ possible values of your test statistic that would result in rejecting H_0

e.g. (in above) say critical region $\bar{x} \leq 48$, $\bar{x} \geq 52$

↑ critical values ↑



(If H_1 1-sided, only get half the picture, one critical value)

④ Look at data & see if estimate is in critical region or not
e.g. if $\bar{x} \geq 52$ or $\bar{x} \leq 48$ in example.

Key: determine critical region → based on how likely it is to end up there accidentally if H_0 true.

Possible outcomes	H_0 true	H_1 true
Accept H_0	✓	Type II Error
Reject H_0 (in favour of H_1)	Type I Error	✓

$\alpha = P(\text{Type I Error})$ = <u>significance level of test</u>

$\beta = P(\text{Type II Error})$

The <u>power</u> of a test is $1-\beta$ (prob. of correctly rejecting H_0)

Type I Error: you're too hard (like finding an innocent (H_0) person guilty (H_1)

Type II Error: you're too cautious (like finding a guilty person (H_1) not guilty (H_0).

<u>Example</u> Sample of 35 observations from Normal pop. with $\sigma = 10$.

Test $H_0 : \mu = 50$

$H_1 : \mu \neq 50$

Critical region is $\bar{x} \geq 52$, $\bar{x} \leq 48$. What is α?

And if true mean μ is really 53, what is β?

<u>Solution</u> $\alpha = P(\text{reject } H_0 \text{ when } H_0 \text{ true})$

$= P(\bar{X} \geq 52 \text{ or } \bar{X} \leq 48 \text{ when } \mu = 50)$

$= P(\bar{X} \geq 52 \text{ when } \mu = 50) + P(\bar{X} \leq 48 \text{ when } \mu = 50)$

$= P\left(Z \geq \frac{52-50}{10/\sqrt{35}}\right) + P\left(Z \leq \frac{48-50}{10/\sqrt{35}}\right)$

$= P(Z \geq 1.18) + P(Z \leq -1.18)$

$= 2(1 - P(Z \leq 1.18)) = 2(1 - 0.881) = \underline{\underline{0.238}}.$

Now if H_0 <u>false</u> we <u>don't</u> get to assume $\mu = 50$ & to find $\beta = P(\text{accept } H_0 \text{ when } H_1 \text{ true})$

$= P(48 \leq \bar{X} \leq 52 \underline{\underline{\text{ when } \mu \neq 50}})$

NOT enough — need true value to compute β i.e. β <u>depends</u> on true value (of μ).

"when $\mu = 53$" instead of "when $\mu \neq 50$"

So if $\mu = 53$,

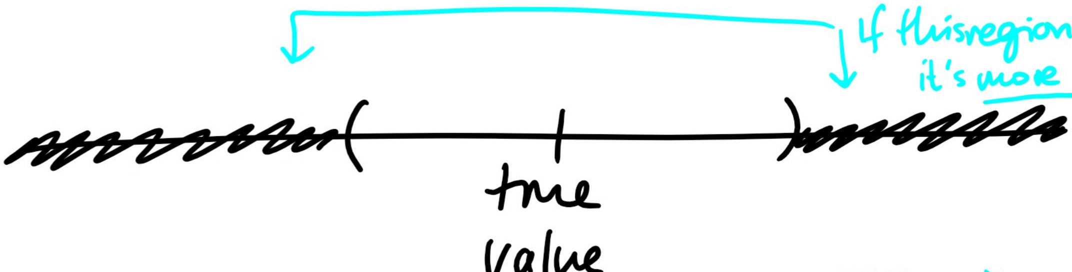
$$\beta = P\left(\frac{48-53}{10/\sqrt{35}} \leq Z \leq \frac{52-53}{10/\sqrt{35}}\right)$$

$$= P(-2.96 \leq Z \leq -0.59) = \underline{\underline{0.274}}.$$

Want to reduce chance of error i.e. make α, β small

↳ so $1-\beta$ = power big!

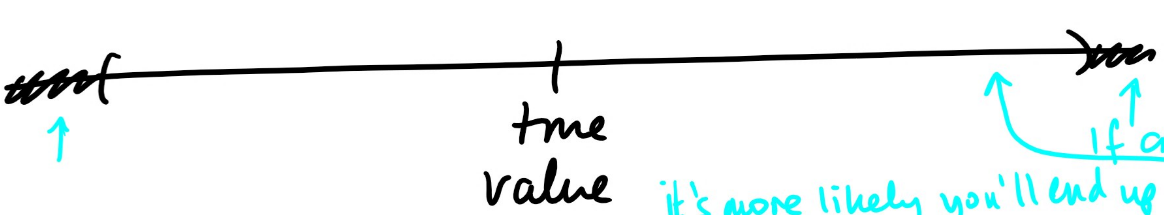
α// ~~smaller~~ bigger (= more likely to have Type I Error) if critical region bigger



If this region is bigger, it's more likely that you'll end up there by mistake

This means that we can only improve one by playing around with the critical region.

β// ~~smaller~~ bigger (= more likely to have Type II Error) if critical region is smaller



If critical region smaller, it's <u>more likely</u> you'll end up outside of it by mistake.

We can increase n to decrease both α & β
(more later)

For now, make a choice:

Fix α small (signifance level) — α typically 0.01, 0.05, 0.1.

$\longrightarrow$ then determine critical region

(β will depend on critical region <u>&</u> on true value of parameter)

<u>Next time</u>: How α and the critical region are related to one another.