3Y03 – PROBABILITY AND STATISTICS FOR ENGINEERING

Last Time HYPOTHESIS TESTING **WS19** **Lecture 25**

H_0 : null hypothesis e.g. $\mu = 50$ ← always equality

H_1 : alternative hypothesis e.g. $\mu \neq 50$, $\mu > 50$, $\mu < 50$

If the value of the test statistic falls in the critical region, then reject H_0 (in favour of H_1). Otherwise, accept H_0.

Type I Error (α): H_0 true but you reject it (too harsh)

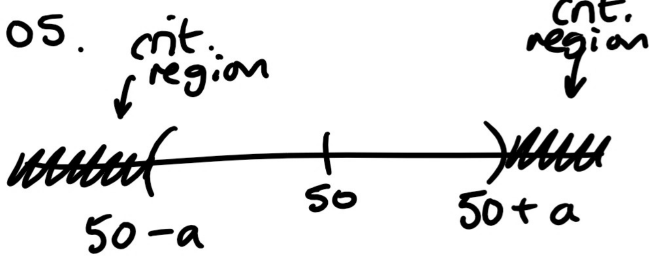
Type II Error (β): H_1 true but you accept H_0 (too cautious)

α = significance level of the test

Example In our setup from last time $n = 35$, $\sigma = 10$, normal pop. Testing $H_0 : \mu = 50$

& $H_1 : \mu \neq 50$

Find the critical region if $\alpha = 0.05$.



Solution

Critical values: $50 \pm a$

Want a to satisfy $0.05 = P(\bar{X} \geq 50 + a \text{ or } \bar{X} \leq 50 - a \text{ when } \mu = 50)$

$\bar{X}$ test stat. for μ

$$= P\left(Z \geq \frac{50 + a - 50}{10/\sqrt{35}}\right) + P\left(Z \leq \frac{50 - a - 50}{10/\sqrt{35}}\right)$$

$$= P\left(Z \geq \frac{a}{10/\sqrt{35}}\right) + P\left(Z \leq \frac{-a}{10/\sqrt{35}}\right)$$

$$= 2\left(1 - P\left(Z \leq \frac{a}{10/\sqrt{35}}\right)\right)$$

$$\Rightarrow \quad P\left(Z \leq \frac{a}{10/\sqrt{35}}\right) = \underline{\underline{1 - \frac{\alpha}{2}}} \quad \text{i.e. } \frac{a}{10/\sqrt{35}} = z_{\alpha/2}$$

Here $\alpha = 0.05$ so $\frac{\alpha}{2} = 0.025$ – look up 0.975

& get $\frac{a}{10/\sqrt{35}} = 1.96$

$\Rightarrow a = 3.313$ so crit region is

$$\bar{x} \leq 46.687, \quad \bar{x} \geq 53.313.$$

P-Values So far "Reject H_0 v. Accept H_0"

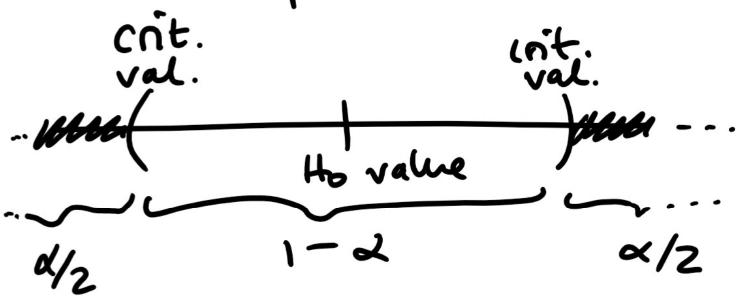
– qualitative.

↓

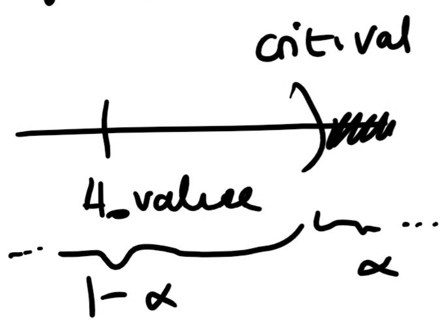
captures how unlikely test stat. value is to occur.

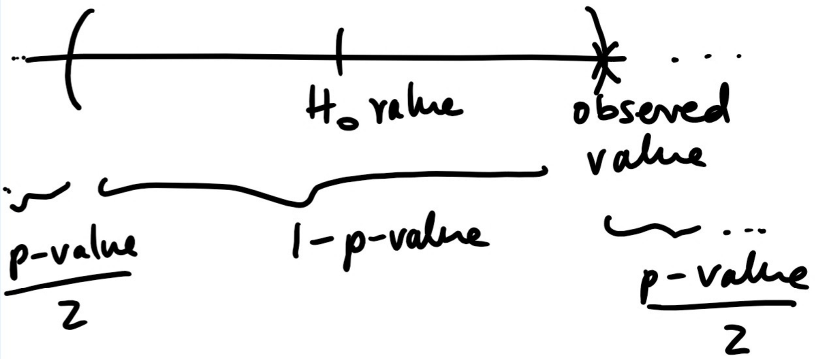
p-value = P(test stat. is at least as <u>extreme</u> (in "direction" of H_1) as observed value)

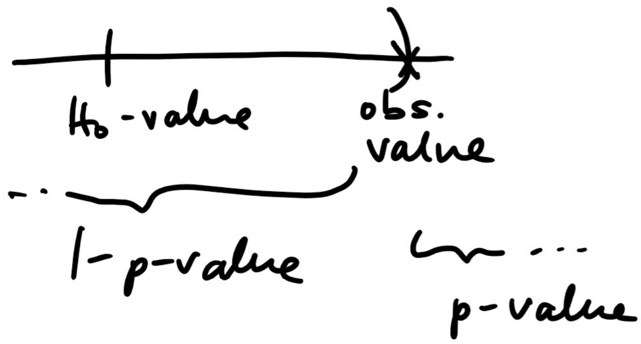
H_1 2 sided



H_1 1-sided







<u>Example</u> (ctd) Normal pop. $n = 35$, $\sigma = 10$, $H_0: \mu = 50$

If $\bar{x} = 54$ what is p-value if (a) $H_1: \mu \neq 50$ (b) $H_1: \mu > 50$?

<u>Solution</u> Which values are "as extreme" as $\bar{x} = 54$?

(a) $\bar{x} \geq 54$ OR $\bar{x} \leq 46$ (b) $\bar{x} \geq 54$

(4 away from H_0-value $\mu = 50$ "in the direction or directions given by H_1")

So in (a) p-value

$$= P(\bar{X} \geq 54 \text{ when } \mu = 50) + P(\bar{X} \leq 46 \text{ when } \mu = 50)$$

$$\vdots$$

$$= P(Z \geq 2.37) + P(Z \leq -2.37)$$

$$= 2(1 - P(Z \leq 2.37)) = \ldots = 0.018.$$

& in (b) p-value $= P(\bar{X} \geq 54 \text{ when } \mu = 50) = 0.009$.

<u>Notice</u> observed value of test. stat. in critical region

$=$ p-value smaller than α

So now our test goes: – find p-value
– compare to α

– If p-value $\leq \alpha$ reject H_0 in favour of H_1
– else accept H_0 – report result along with p-value

<u>Notice</u> Close relationship between C.I.s & H.Ts

If $H_0 : \theta = \theta_0 \leftarrow$ #, "null value"
$H_1 : \theta \neq \theta_0$

C.I. for θ : centred on observed value $\hat{\theta}$

$100(1-\alpha)\%$ C.I. : $100(1-\alpha)\%$ confident that true value of θ is in C.I.

H.T. : - region centred on H_0 - value θ_0
- reject H_0 if observed value $\hat{\theta}$ is <u>outside</u> the interval.

But "Margin of Error" is same in both cases if α value is same. → This is important to note because sometimes on the formula sheet the Margin of Error formula is only listed in one of the two spots.

9.2 Hypothesis Test on the Mean of a Normal Distribution, Variance Known

$H_0 : \mu = \mu_0$

$H_1 :$ (I) $\mu \neq \mu_0$ (II) $\mu > \mu_0$ (III) $\mu < \mu_0$

Since we standardize every time, we think of $Z_0 = \dfrac{\overline{X}-\mu_0}{\sigma/\sqrt{n}}$ as the test stat.

Based on H_0 assumption, we are assuming

$$Z_0 \sim N(0,1).$$

So this is called a Z-test:

At significance level α: ① Compute

$$z_0 = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$$

from data.

② Reject H_0 if $\underbrace{\text{p-value of } z_0 \text{ is } \le \alpha.}_{\text{3 cases for what this means:}}$

<u>Case (I)</u>

$H_1 : \mu \neq \mu_0$

$$\text{p-value} = \underbrace{P\left(Z_0 \geq \left|\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}\right|\right) + P\left(Z_0 \leq -\left|\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}\right|\right)}_{2\,\underbrace{P\left(Z_0 \geq \left|\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}\right|\right)}}$$

$\Rightarrow$ Reject H_0 if $\alpha \geq 2(\quad)$

$$\Rightarrow \text{ if } \frac{\alpha}{2} \geq P\left(Z_0 \geq \overbrace{\left|\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}\right|}^{z_0}\right)$$

$$\Rightarrow z_{\alpha/2} \leq |z_0|$$

So critical region is $z_0 \leq -Z_{\frac{\alpha}{2}}$ or $z_0 \geq Z_{\frac{\alpha}{2}}$.

T.B.C. (remaining cases in the next lecture)