# 3Y03 - PROBABILITY AND STATISTICS FOR ENGINEERING

WS19 Lecture 27

## Yesterday

Z-test for mean of Normal population, variance known ($\sigma^2$)

Test stat. $Z_0 = \dfrac{\bar{X} - \mu_0}{(\sigma/\sqrt{n})}$ $\sim N(0,1)$ if $H_0$ true

$H_0 : \mu = \mu_0$ $\quad H_1$ :

| | (I) $\mu \neq \mu_0$ | (II) $\mu > \mu_0$ | (III) $\mu < \mu_0$ |
|---|---|---|---|
| CRITICAL REGION: | $\lvert z_0 \rvert > z_{\alpha/2}$ | $z_0 > z_\alpha$ | $z_0 < -z_\alpha$ |
| P-VALUE: | $2P(Z_0 > \lvert z_0 \rvert)$ | $P(Z_0 > z_0)$ | $P(Z_0 < z_0)$ |
| TO FIND $n$ IN TERMS OF $\beta$ : ($\mu = \mu_0 \pm \delta$) | $n \approx \dfrac{(z_{\alpha/2} + z_\beta)^2 \sigma^2}{\delta^2}$ | $n \approx \dfrac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta^2}$ | |

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## 9.3 Test on Mean of Normal Pop., Variance Unknown

sample size big $[n \geq 40]$ — use everything as in variance known case but replace $\sigma^2$ with $S^2$

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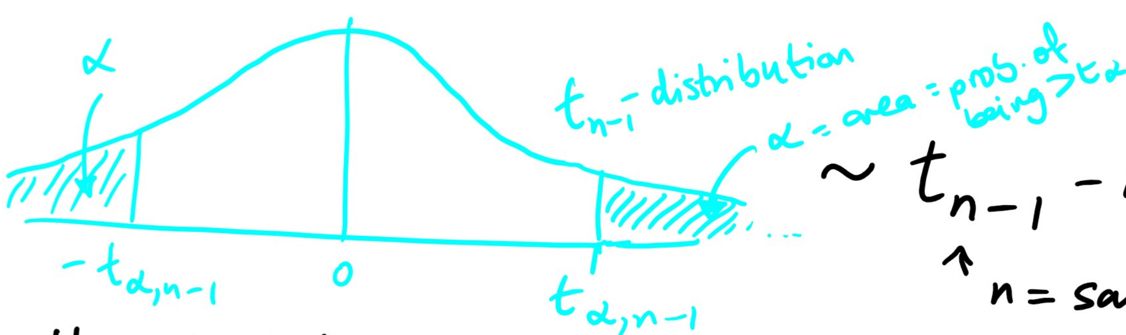
sample size small — test. stat. $\dfrac{\bar{X} - \mu_0}{S/\sqrt{n}} = T_0$




$\sim t_{n-1}$ - distribution

n = sample size

$H_0 : \mu = \mu_0$

$H_1$ : (I) $\mu \neq \mu_0$ | (II) $\mu > \mu_0$ | (III) $\mu < \mu_0$

CRITICAL REGIONS: $\lvert t_0 \rvert > t_{\frac{\alpha}{2}, n-1}$ | $t_0 > t_{\alpha, n-1}$ | $t_0 < -t_{\alpha, n-1}$

P-VALUE : $2P(T_0 > |t_0|)$ | $P(T_0 > t_0)$ $P(T_0 < t_0)$

We call this a t-test because our choice of test statistic, $T_0$, has one of the t-distributions.

<u>Example</u> For a hypothesis test on mean $\mu$ of Normal pop., unknown $\sigma^2$, approximate the p-value when

(a) $H_0 : \mu = \mu_0$
$H_1 : \mu \neq \mu_0$
$t_0 = 2.537$
$n = 10$

(b) $H_0 : \mu = \mu_0$
$H_1 : \mu > \mu_0$
$t_0 = 1.863$
$n = 16$.

(a) $n = 10$

$n - 1 = 9 \rightarrow$ row 9 in t-table

$t_0 = 2.537 \rightarrow$ lies in range corresponding to

This tells us the range of possible values for $P(T_0 > |t_0|)$. $\rightarrow (0.01, 0.025)$ (from top "2" row)

So p-value (2-sided case) lies in $(0.02, 0.05)$.

(b) $n = 16$

$n - 1 = 15 \rightarrow$ row 15

$t_0 = 1.863 \rightarrow$ lies in range corresponding to

The range of possible values for $P(T_0 > t_0)$ $\rightarrow (0.025, 0.05)$

So p-value (1-sided case) lies in $(0.025, 0.05)$.

# 9.5 Tests on Population Proportion

Recall : $\hat{p} = \frac{X}{n} = \frac{\#\text{ observations in some category}}{\#\text{ observations}}$

estimator of proportion $p$ where $X \sim Bin(n, p)$

By Normal approx. to Binomial, $X \sim N(np, np(1-p))$

(if $np > 5$, $n(1-p) > 5$)

Test $H_0 : p = p_0$

$H_1 :$ (I) $p \neq p_0$ (II) $p > p_0$ (III) $p < p_0$

If $H_0$ is true, then $X \sim N(np_0, np_0(1-p_0))$

so $\hat{p} = \frac{X}{n} \sim N\left(p_0, \frac{p_0(1-p_0)}{n}\right)$

So $Z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \sim N(0,1)$

There is disagreement amongst statisticians about whether or not there should be continuity corrections done in computing probabilities here. The textbook does not use them in this context, so we will also not use them & keep it simple!

We have to compute $z_0$ using this & then:

| | $H_1$ | p-value | critical region |
|---|---|---|---|
| (I) | $p \neq p_0$ | $2P(Z_0 > \lvert z_0 \rvert)$ | $\lvert z_0 \rvert > z_{\alpha/2}$ |
| (II) | $p > p_0$ | $P(Z_0 > z_0)$ | $z_0 > z_\alpha$ |
| (III) | $p < p_0$ | $P(Z_0 < z_0)$ | $z_0 < -z_\alpha$ |

<u>Example</u> A company claims 90% of people are satisfied with their gizmo. Of a random sample of 80 people, 65 are satisfied. At significance level $\alpha = 0.01$, does the evidence support the company's claim?

<u>Solution</u> $\hat{p} = \frac{65}{80} = 0.825$

$H_0 : p = 0.9$

$H_1 : p < 0.9$

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.825 - 0.9}{\sqrt{\frac{0.9(0.1)}{80}}} = -2.24$$

Compare $z_0$ to $-z_\alpha = -z_{0.01} = -2.33$

Since $\overbrace{-2.24}^{z_0} > \overbrace{-2.33}^{-z_\alpha}$, $z_0$ not in critical region, so do not reject $H_0$.

$z_0 < -z_\alpha$ in case (III).