3Y03 - PROBABILITY AND STATISTICS FOR ENGINEERING

WS19 Lecture 3

Last time

Events Collections of possible outcomes i.e. subsets of the sample space

Multiplication Rule

Outcome after k steps; n_i ways of completing step i.

Total # possible outcomes $= n_1 \times n_2 \times \ldots \times n_k$.

Permutations & Combinations

A k-Permutation of n distinct objects is an ordered arrangement of some k of those objects.

Recall: licence plates letters _ _ _ _ (4 letters) $= k$

pool of 26 letters $= n$

There were $\dfrac{26!}{(26-4)!}$.

How many k-Permutations of n objects are there?

n choices for 1st object

$n-1$ choices for 2nd object

$\vdots$

$n-k+1$ choices for kth object

So multiplication rule tells us there are

$$n \times (n-1) \times \dots \times (n-k+1) \text{ choices}$$

$$= \frac{n!}{(n-k)!} =: P^n_k$$

Recall: 3 signals, arrive (A) or not (N)

\# ways we get exactly 2 signals arriving: AAN, ANA, NAA

How many ways can we permute the order of A, A, N

$A_1 \quad A_2$

Suppose we temporarily label the As

These two temporarily look different:

$A_1 A_2 N$
$A_2 A_1 N$

but are really same AAN – so should only be counted once

Permutations Rule

If we have n objects of r different types with

n_1 of type 1
n_2 of type 2
$\vdots$
n_r of type r,

(Notice: $n_1 + \dots + n_r = n$)

then the # of ~~x~~-permultions of our n objects is

Let's just call it a "permutation" – without any prefix – if we organize all n objects into a list.

$$\frac{n!}{n_1!\, n_2! \cdots n_r!}$$

Idea: if all objects are of different types we have $n!$ arrangements. Focus on type 1 if we reorder these & keep everything else fixed, there are $n_1!$ ways of doing that. Repeat for type 2 etc.

We then divide out by $n_1!, n_2!, \ldots, n_k!$ in order not to count indistinguishable lists more than once.

So for our example with A, A, N

$$\left.\begin{aligned} n &= 3 \\ n_1 &= 2 \\ n_2 &= 1 \end{aligned}\right\} \rightarrow \frac{3!}{2!\,1!}$$

$$= \frac{6}{2.1} = 3$$

<u>Example</u> Suppose a hospital O.R. has to schedule 2 hip surgeries, 3 knee surgeries & 1 shoulder surgery.

(a) How many possible schedules are there (if all that matters is type of surgery)?

(b) How many are there, if knee surgeries at start & end?

<u>Solution</u> (a) If person mattered (so can distinguish different surgeries of same type)

then answer $= P^6_6 = 6!$

But with repeated instances of same type we have $6! \big/ 2!3!1! = \cancel{420}$ 60

(b) $\underline{K}\ \underbrace{\_\ \_\ \_\ \_}\ \underline{K}$ $\qquad \dfrac{4!}{2!\,1!\,1!} = 12.$

Put 2H, 1K, 1S

Combinations

Now principally care about making choices where order doesn't matter.

Example 20 people are potential jurors from which 12 must be chosen for a jury. How many ways?

If we care about the order in which they sit, this is $P^{20}_{12} = \dfrac{20!}{\cancel{12!}\ 8!} = \cancel{5{,}079{,}110{,}400.}$ 60,339,831,552,000

If order doesn't matter then answer given by:

r- Combinations

of subsets of r elements that can be selected from n ~~elements~~ objects is given by

$$\frac{n!}{r!(n-r)!} = C^n_r = \binom{n}{r}$$

"n choose r" Binomial coefficient

i.e. # of juries of 12 from 20 $= C^{20}_{12} = \binom{20}{12} = \frac{20!}{12!\,8!} = 125{,}970$

Example Now suppose of 20 potential jurors, 15 are men and 5 are women.

How many poss. juries of 12 are there with exactly 2 women?

Solution By multiplication rule $\binom{15}{10} \times \binom{5}{2}$.

ways to choose req. # of men

of ways to choose req. # of women

Example Batch of 100 semiconductor chips.
75 conform to standards
25 do not.

Select 3 to test.

(a) # different test samples of 3? $= \binom{100}{3}$

(b) # samples with exactly 1 non-conform. chip?

$$= \binom{75}{2} \times \binom{25}{1}$$

ways to choose the 2 conforming chips

ways to choose the 1 non-conf. chip.

(c) # samples with at least 1 non-conf. chip?

(1) exactly 1 or exactly 2 or exactly 3

$$= \binom{75}{2} \times \binom{25}{1} + = \binom{75}{1} \times \binom{25}{2} + = \binom{75}{0} \times \binom{25}{3}$$

(2) we want total # samples − # samples with 0 non-conf.

$$\binom{100}{3} - \binom{75}{3} \times \binom{25}{0}.$$

CHECK that these give the same answer!