3Y03 - Probability and Statistics for Engineering

WS19 Lecture 4

Last time

Permutations & Combinations

- $n = n_1 + n_2 + \ldots + n_k$ objects, n_i of type i, can be arranged in an ordered list in $\dfrac{n!}{n_1! n_2! \cdots n_k!}$ ways.
- r objects can be chosen out of n objects in

$$C^n_r = \binom{n}{r} = \frac{n!}{r!(n-r)!} \text{ ways.}$$

2-2 Probability

- function assigned to events of sample space
- represents belief in what the results of an "experiment" are likely to be

- subjective : best guess e.g. betting odds
  one-time events

- objective : relative frequency of events in repetitions of same experiment "settle down in long run"

To simplify matters: for now work with discrete sample spaces.

Suppose S finite: $S = \{ s_1, \ldots, s_N \}$

N could be v. large

Assign <u>weights</u> to outcomes

i.e. non-negative # p_i assigned to s_i

where $\displaystyle\sum_{i=1}^{N} p_i = 1$ $\leftarrow$ notice $0 \leq p_i \leq 1$

For an event E, the probability of E

$$P(E) = \sum_{\{i \mid s_i \in E\}} p_i$$

– i.e. add up all the weights p_i for all outcomes s_i in E.

If $P(E) = 1$, E is certain to happen.

If $P(E) = 0$, E is certain not to happen.

<u>Example</u> $S = \{a, b, c\}$ with weights 0.1, 0.5 and 0.4.

If $A = \{a, c\}$, $B = \{b, c\}$ then

$$P(A) = 0.1 + 0.4 = 0.5$$
$$P(B) = 0.5 + 0.4 = 0.9$$
$$P(A \cap B) = P(\{c\}) = 0.4.$$

Simplest situation — very often assumed — is that all outcomes are "equally likely" (we also say "random") i.e. same weight assigned to all outcomes

We know all p_is same and $\sum_{i=1}^{N} p_i = 1$

So $p_i = \frac{1}{N}$

In that case $P(E) = \frac{1}{N}(|E|)$

elements in E.

<u>Example</u> 100 chips

75 conform to standards

25 non-conforming

3 selected for testing. ← Assume all selections equally likely

E = choose exactly 2 non-conforming

$$P(E) = \frac{|E|}{N} = \frac{\binom{25}{2} \times \binom{75}{1}}{\binom{100}{3}} \simeq 0.14.$$

We could introduce a variable X to count # non-conforming chips we choose amongst the 3

The above event E is the event $X = 2$

X is called a <u>random variable</u>.

A <u>probability distribution</u> tells us the prob. associated with each value of a random variable X.

e.g. here

x	$P(X=x)$	
0	0.42	$= \binom{75}{3} / \binom{100}{3}$
1	0.43	
2	0.14	← above
3	0.01	
	1	← should add up to 1

$P(S) = 1$.

Axioms of Probability — rules

— allow us to conclude info. about probabilities based on 'known' info.

(1) $P(S) = 1$

(i.e. prob. of something happening is certain)

(2) $0 \leq P(E) \leq 1$

(3) If $E_1 \cap E_2 = \phi$ (i.e. E_1, E_2 are mutually exclusive)

then $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

↳ Notice this tells us $P(\phi) = 0$

and $P(E^c) = 1 - P(E)$

$= E'$ 2 notations for the same thing — the "complement" of E

$(E \cap E^c = \phi$
$E \cup E^c = S)$

Notice also: can extend (3) to as many events as you like, as long as they are "pairwise mutually exclusive" i.e. if $E_1, \ldots, E_n$ (or $E_1, \ldots$)

all pairwise mutually excluse i.e.

$E_i \cap E_j = \phi$ for any pair i, j

then $P(E_1 \cup E_2 \cup \ldots \cup E_n) = \sum_{i=1}^{n} P(E_i)$

$\left(P(E_1 \cup E_2 \cup \ldots) = \sum_{i=1}^{\infty} P(E_i). \right)$

But what is $P(E_1 \cup E_2)$ if E_1, E_2 NOT mutually exclusive?

(some outcome in <u>both</u> E_1 & E_2).

<u>Addition Rule</u> $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

<u>Example</u>

	Passed	Failed	TOTAL
CO1	123	48	171
CO2	115	32	147
TOTAL	238	80	318

E_1 : randomly chosen student in CO2

E_2 : " " " passed

(a) $P(E_1) = \frac{147}{318}$ (b) $P(E_2) = \frac{238}{318}$ (c) $P(E_1 \cap E_2) = 115/318$

(d) $$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$
$$= \frac{147}{318} + \frac{238}{318} - \frac{115}{318} = \frac{270}{318}.$$