3Y03 – PROBABILITY AND STATISTICS FOR ENGINEERING

WS19 **Lecture 5**

Yesterday

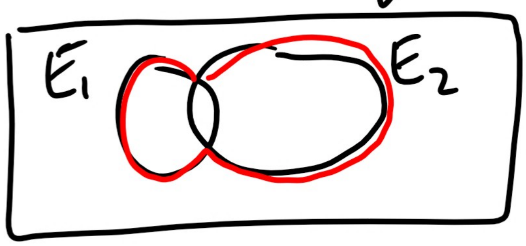
PROBABILITY $P(E)$ ← event (subset of S)

Axioms: (1) $P(S) = 1$ (2) $0 \leq P(E) \leq 1$

(3) If $E_1 \cap E_2 = \phi$, then $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

(by extension, if $E_i \cap E_j = \phi$ $(i \neq j)$, then $P(\bigcup_{i=1}^{n} E_i) = \sum_{i=1}^{n} P(E_i)$)

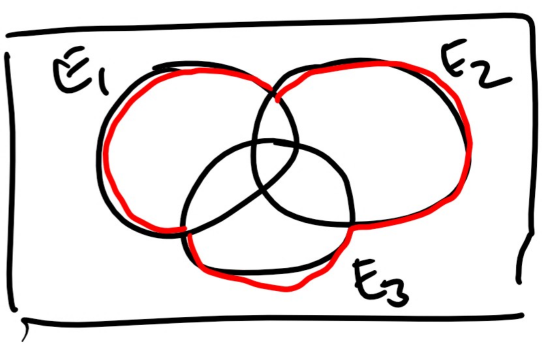
Addition Rule $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$



↑ 2 events

You will find both formulas on the "formula sheet" for tests/exams

3 events ↓



$$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_1 \cap E_3) - P(E_2 \cap E_3) + P(E_1 \cap E_2 \cap E_3)$$

Example Passwords of 8 characters can have 26 upper case & 26 lower case letters & 10 #s

Randomly chosen password (all passwords

equally likely). What is the probability it has 2 at start <u>or</u> odd # at end <u>or</u> K in 5th spot?

<u>Solution</u>

E_1 : 2 at start $\rightarrow P(E_1) = \frac{|E_1|}{N}$

E_2 : odd # end $\rightarrow P(E_2) = |E_2|/N$

E_3 : K in 5th spot $\rightarrow P(E_3) = |E_3|/N$

$$\text{So } P(E_1 \cup E_2 \cup E_3) = \underbrace{\frac{62^7}{62^8}}_{P(E_1)} + \underbrace{\frac{62^7 \times 5}{62^8}}_{P(E_2)} + \underbrace{\frac{62^7}{62^8}}_{P(E_3)} - \underbrace{\frac{62^6 \times 5}{62^8}}_{P(E_1 \cap E_2)} - \underbrace{\frac{62^6}{62^8}}_{P(E_1 \cap E_3)} - \underbrace{\frac{62^6 \times 5}{62^8}}_{P(E_2 \cap E_3)} + \underbrace{\frac{62^5 \times 5}{62^8}}_{P(E_1 \cap E_2 \cap E_3)} = \cdots \doteq 0.11.$$

$$P(E_1 \cap E_2) = \frac{62^6 \times 5}{62^8}$$

When all outcomes equally likely we have that $P(E_1 \cup E_2 \cup E_3)$

$$= \frac{|E_1| + |E_2| + |E_3| - |E_1 \cap E_2| - |E_2 \cap E_3| - |E_1 \cap E_3| + |E_1 \cap E_2 \cap E_3|}{N}$$

<u>Conditional Probability</u>

- update probabilities based on new info.

$P(A \mid B)$ = the <u>conditional</u> prob. of event A <u>given</u> B.

i.e. knowing B occurred.

<u>Example</u> Failure of heating systems:

E: electrical failure →

G: gas leak ↳

	Y	N	TOTAL
Y	32	5	37
N	21	12	33
TOTAL	53	17	70

What is $P(G) = 37/70$ $\qquad$ $P(E \cap G) = \frac{32}{70}$

$P(G \mid E) = 32/53$

$P(E \mid G) = 32/37$

<u>Rule</u>: $$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

So above $P(E \mid G) = \dfrac{32/70}{37/70} = \dfrac{32}{37}.$

When equal. prob. $P(A|B) = \dfrac{\#\text{ outcomes in } \cancel{B}\, A \text{ amongst outcomes in } \cancel{A}\, B}{\#\text{ outcomes in } B}$

Some of the outcomes of B (the ones also in A)

all of the outcomes in B

<u>Example</u> 200 chips, 10 defective, sample of 2 chosen without replacement.

E_1 : {first defective}

E_2 : {second defective}

(a) $P(E_2 | E_1) = \dfrac{9}{199}$

(b) $P(E_1 \cap E_2) =$

$P(E_2 | E_1)\, P(E_1)$

$= \dfrac{9}{199} \times \dfrac{10}{200}$

Can rearrange above rule:

$P(A \cap B) =$

$P(A|B)P(B)$

$= P(B|A)P(A)$

(c) P (both not defective) $= P(E_1^c \cap E_2^c)$

$= P(E_2^c | E_1^c) P(E_1^c)$

(Exercise) $= \dfrac{189}{199} \times \dfrac{190}{200}$.

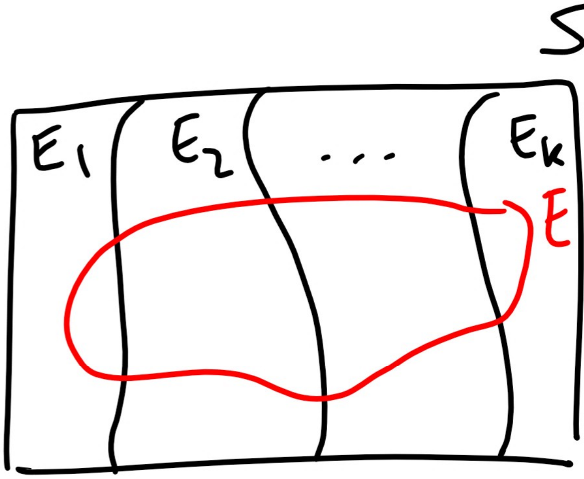
If you can set up problem as a chain of dependencies & know prob. of each step

<u>given</u> previous steps, can simplify calculation

More sophisticated version:

E - event.

$E_1, E_2, \ldots, E_k$ - exhaustive list of "conditions"



- mutually exclusive

i.e. partition sample space S

$(S = E_1 \cup E_2 \cup \ldots \cup E_k)$

$E_i \cap E_j = \phi \quad i \neq j$

Then $P(E) = P(E \cap E_1) + \ldots + P(E \cap E_k)$

$$P(E) = P(E|E_1)P(E_1) + \ldots + P(E|E_k)P(E_k)$$

TOTAL PROBABILITY RULE.

<u>Example</u> Manufacture semi-conductors & know prob. of failure depending on contamination level (Low, Medium, High)

exhaustive list

i.e. We know ↓

F = failure $P(F \mid H) = 0.2$

$P(F \mid M) = 0.05$

$P(F \mid L) = 0.0001$

We also know $P(H) = 0.1$, $P(M) = 0.3$, $P(L) = 0.6$ } sum to 1 (exhaustive list of possibilities)

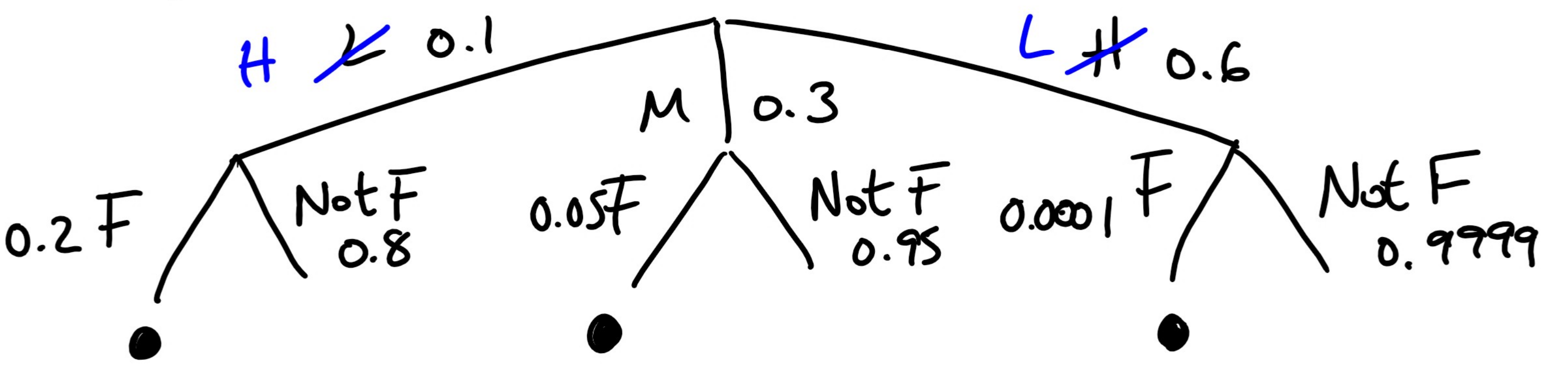
Find $P(F) = P(F \mid L)P(L) + P(F \mid M)P(M) + P(F \mid H)P(H)$

$= 0.2 \times 0.1 + 0.05 \times 0.3 + 0.0001 \times 0.6$

$= 0.03506.$

Apologies, I wrote the terms in a different order

Can represent this as a tree:



P(F) = find F at bottom of tree (•) & multiply along branches.