3Y03 - PROBABILITY AND STATISTICS FOR ENGINEERING

WS19 Lecture 7

PROBABILITY SO FAR...

- $P(S) = 1$, $P(\phi) = 0$.
- $P(E') = 1 - P(E)$

$$\begin{cases} \bullet\ P(E_1 \cup E_2) = P(E_1) + P(E_2), \text{ IF } E_1 \cap E_2 = \phi \\ \bullet\ P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2), \text{ IN GENERAL} \end{cases}$$

Addition Rule

$$\begin{cases} \bullet\ P(E_1 \cap E_2) = P(E_1 | E_2) P(E_2) \\ \bullet\ P(E_1 \cap E_2) = P(E_1) P(E_2), \text{ IF } E_1, E_2 \text{ INDEPENDENT} \end{cases}$$

(Last time)

Example Video game with 5 opponents (in sequence). Game ends when defeated or run out of opponents. 70% chance of beating each opponent & each matchup/result independent of others. Let E_i be event: beat player i $(i=1,\dots,5)$

$$P(\text{beat all } 5) = P(E_1 \cap E_2 \cap E_3 \cap E_4 \cap E_5)$$
$$= P(E_1) P(E_2) \dots P(E_5)$$
$$= (0.7)^5 = 0.16807.$$

$$P(\text{beat at least } 3) = P(E_1 \cap E_2 \cap E_3)$$
$$= P(E_1) P(E_2) P(E_3) = (0.7)^3$$
$$= 0.343$$

↓

$$P\Big(\overbrace{(E_1 \cap E_2 \cap E_3 \cap E_4')}^{A} \cup \overbrace{(E_1 \cap E_2 \cap E_3 \cap E_4 \cap E_5')}^{B}$$

$$\cup \underbrace{(E_1 \cap E_2 \cap E_3 \cap E_4 \cap E_5)}_{C}\Big)$$

e.g.

$$\boxed{\begin{aligned} P(A) &= P(E_1)P(E_2)P(E_3)P(E_4') \\ &= (0.7)^3(0.3). \end{aligned}}$$

↳ gives same answer 0.343.

2.8 Random Variables

→ Represent possible numerical outcomes of experiment

→ Capital letter e.g. X or Y

→ take values at small letters e.g. x or y

Just as with outcomes in sample space:

Discrete Random Variables (r.v.) : take values in a finite (or countably infinite) range

Continuous Random Variables : take values in a range containing an interval

Example Play video game, 5 opponents in sequence, stop after losing or no more opponents. Let X = # wins

X has range $\{0, 1, 2, 3, 4, 5\}$

– discrete r.v.

Can write down prob. X takes each value:

$$P(E_1') = P(X=0) = 0.3$$

$$P(E_1 \cap E_2') = P(X=1) = 0.7 \times 0.3 = 0.21$$

$$P(E_1 \cap E_2 \cap E_3') = P(X=2) = (0.7)^2 \times 0.3 = 0.14$$

$$P(E_1 \cap E_2 \cap E_3 \cap E_4') = P(X=3) = (0.7)^3 \times 0.3 = 0.1029$$

$$P(E_1 \cap \ldots \cap E_4 \cap E_5') = P(X=4) = (0.7)^4 \times 0.3 = 0.07203$$

$$P(E_1 \cap \ldots \cap E_5) = P(X=5) = (0.7)^5 = 0.16807$$

This is the <u>probability distribution</u> of X

Sum of probabilities $= 1$.

Shift of focus: from now on events encoded by random variables

3.2 Probability Mass Function

↳ for <u>discrete</u> r.v.s.

For discrete r.v. X taking values in range

$$R_X = \{x_1, x_2, \ldots, x_n\} \quad \text{OR}$$

$$R_X = \{x_1, x_2, \ldots\}$$

a <u>probability mass function</u> (pmf) for X is a function $f(x)$ such that:

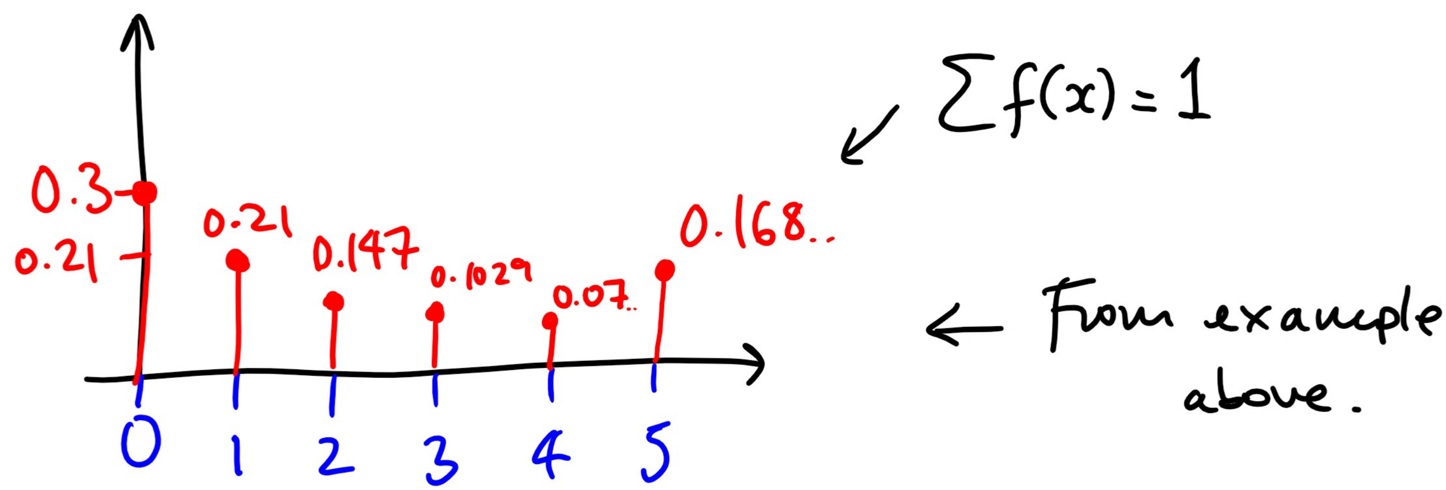
(1) $P(X = x_i) = f(x_i)$

(2) $f(x_i) \geq 0$

(3) $\sum_{i=1}^{n} f(x_i) = 1$ or $\sum_{i=1}^{\infty} f(x_i) = 1$

(as appropriate).

We can plot / visualize a pmf:



<u>Facts</u> Axiom 3 $P(E_1 \cup E_2) = P(E_1) + P(E_2)$ <u>if</u> $E_1 \cap E_2 = \emptyset$ tells us

$$P(X \in A) = \sum_{x \in A} f(x)$$

set of values in range of X

Why? Notice, the events $X = x_1$ and $X = x_2$ are mutually exclusive if $x_1 \neq x_2$.

Example Inspection of 2 machines.
Prob. that Machine 1 meets specs is 0.98 & prob. for Machine 2 is 0.95.
The failures of the machines are independent events.

Let X = # machines meeting specifications.

$$P(X=0) = (1-0.98)(1-0.95) = (0.02)(0.05) = 0.001$$
$= f(0)$

$$P(X=1) = (0.98)(0.05) + (0.02)(0.95) = 0.068$$
$= f(1)$

$$P(X=2) = (0.98)(0.95) = 0.931$$
$= f(2)$

sum to 1.

3.3 Cumulative distribution function

pmf: $f(x) = P(X = x)$

cdf: $F(x) = P(X \leq x) = \sum_{x_i \leq x} f(x_i)$.

<u>Example</u> Video game example:

x	0	1	2	3	4	5
$f(x)$	0.3	0.21	0.147	0.1029	0.072	0.168

From this can find cdf $F(x)$:

e.g. $P(X \leq -1) = 0$

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 0.3 & \text{if } 0 \leq x < 1 \\ 0.51 & \text{if } 1 \leq x < 2 \\ 0.657 & \text{if } 2 \leq x < 3 \\ 0.749 & \text{if } 3 \leq x < 4 \\ 0.8319 & \text{if } 4 \leq x < 5 \\ 1 & \text{if } 5 \leq x \end{cases}.$$

Notice that for the cdf we take into account all x-values