Math 2R03 Final Exam Info Sheet

The purpose of this handout is to help you study by listing the concepts, definitions, and results you will need to know for the final exam.

Final Exam Information. The final will be on Friday, December 13. The final will be two hours and 30 minutes long. The time and location can be found on MOSAIC or A2L closer to the exam date. Please remember to bring your **Student Card** with you.

The final exam will have 14 questions. You will have to answer all of Questions 1-7. You will have to do 5 of Questions 8-13. Question 14 is a bonus questions. The final exam is out of 45 points. You will be allowed to bring one 8.5×11 "cheat sheet", which you can write anything you want, using both sides. You may also use the standard McMaster calculator (Casio FX-991 MS or MS Plus).

Material Covered. The final exam is cumulative, although there will be more of an emphasis on the material since the last midterm. For a breakdown on what you need from Lectures 1-21, see the review sheets for Midterms 1 and 2. Below is a breakdown of what you will need to know from Lectures 22-35. Note that when you are learning definitions, it is good to know an example of that definition, and an example of an object that does not satisfy the definition.

Section 8.A Know Theorems 8.1, 8.2, 8.3, and 8.4 which all describe how the null space of T^i changes as i increases. Know what is meant by a generalized eigenvector and generalized eigenspace. Know Theorems 8.9, 8.11 and 8.12. Know what is meant by a nilpotent operator and Theorems 8.17 and 8.18.

Section 8.B Know Theorem 8.22 and how to use it to decompose a vector space into generalized eigenspaces. Know what is meant by multiplicity, and the connection to algebraic multiplicity and geometric multiplicity. Know what the characteristic polynomial of an operator is. Know Theorem 8.29, the Cayley-Hamilton theorem. Know what is meant by a block diagonal matrix and know Theorem 8.37.

Section 6.A Know the definition of an inner product (6.2) and be able to determine if a given function is an inner product. Know what an inner product space is, and know the properties of an inner product (e.g. Theorem 6.6). Know what we mean by a norm, and orthogonal vectors. Know the Pythagorean Theorem, Cauchy-Schwarz inequality, the triangle inequaltiy, and the parallelogram equality.

Section 6.B Know what it means for a collection of vectors to be orthonormal, and know what an orthonormal basis is. Important properties of orthonormal sets are in Theorems 6.24, 6.25, and 6.30. Know how to apply the Gram-Schmidt procedure. Know 6.35, 6.36, and Schur's Theorem. Know what we mean by a linear functional, and know the Riesz Representation Theorem. In particular, be able to find the vector u in this theorem.

Section 6.C Know what is meant by the orthogonal complement of a subspace. Know some of the properties of orthogonal complements (e.g. Theorem 6.48, 6.49, 6.52). Know what we mean by the orthogonal projection. Know Theorem 6.61.

Section 7.A Know what an adjoint of an operator is. Know the properties of the adjoint (Theorem 7.5,7.6). Know what we mean by the conjugate transpose, and how to find the matrix of the adjoint. Know what a self-adjoint operator is, and know some of its properties (e.g Theorem 7.12). You will not be tested on 7.13, 7.14, or 7.16. Know what a normal operator is and some of its properties (e.g., 7.20, 7.21, and 7.22). You do not need to know 7.23.

Section 7.B Know the Complex Spectral Theorem and the Real Spectral Theorem. For both theorems, you need to know how to apply the statements. You will not be tested on their proofs.

Section 5.D You will not be tested on the Gershgorin Disk Theorem.

If you have questions, please feel free to email me. The TA and I will also schedule a review session and office hours during the exam session. Please see Avenue-to-Learn for more details. Good luck!