MATH 2R03 (THEORY OF LINEAR ALGEBRA)
DUE: SEPTEMBER 13, 2024

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## HOMEWORK ASSIGNMENT 1

All of the questions from Part A will be graded. One of the questions from Part B will be graded in detail, while the other will be marked for completion. Assignments will be submitted via *Crowdmark*. You will be graded on your solution *and* how well you write your proof.

**Part A.** [Short Questions; 4pts]

**Exercise 1**. Let $V$ be a vector space over a field $F$. Show that for all $x \in V$ there is a unique $y$ such that $x + 2024y = 0$.

**Exercise 2**. Show that the set $S = \{(x_1, x_2) \mid x_1, x_2 \in \mathbb{R}, x_1 \leq 0, x_2 \geq 0\}$ is not a real vector space.

**Part B.** [Proof Questions; 6pts]

**Exercise 3**. Let $V = F^\infty$ and let

$$W = \{(x_1, \ldots, x_{2023}, 0, x_{2025}, x_{2026}, \ldots) \mid x_i \in F\} \subseteq V.$$

That is, $W$ consists of all the elements of $F^\infty$ whose $2024^{th}$ coordinate is zero. Prove that $W$ is a subspace of $V$.

**Exercise 4**. Consider the subspaces $U = \{(2024x, 0) \mid x \in \mathbb{R}\}$ and $V = \{(y, y) \mid y \in \mathbb{R}\}$ of $\mathbb{R}^2$. Prove that $\mathbb{R}^2 = U + V$.

*Hint.* There are two things you need to show: (a) $\mathbb{R}^2 \subseteq U + V$ and (b) $U + V \subseteq \mathbb{R}^2$. (You do not need to prove that $U$ and $V$ are subspaces, but you should convince yourself that this is true.)

**Additional Suggested Problems.** [Not graded]

Problems 1.A # 10, 11, 15, 1.B # 1, 2, 3 1.C # 6, 10, 23