Math 2R03 (Theory of Linear Algebra)
Due: September 20, 2024

Homework Assignment 2

All of the questions from Part A will be graded. One of the questions from Part B will be graded in detail, while the other will be marked for completion. Assignments will be submitted via *Crowdmark*. You will be graded on your solution *and* how well you write your proof.

Part A. [Short Questions; 4pts]

Exercise 1. Suppose that $V = \operatorname{span}(v_1, v_2, v_3)$. Prove that the list

$$v_1 + v_2 + v_3, v_2, 2024v_3$$

also spans V.

Exercise 2. Consider the subspaces $U = \{(2024x, 0) \mid x \in \mathbb{R}\}$ and $V = \{(y, y) \mid y \in \mathbb{R}\}$ of $\mathbb{R}^2$. In Homework 1, you proved that $\mathbb{R}^2 = U + V$. Show that in fact $\mathbb{R}^2 = U \oplus V$.

Part B. [Proof Questions; 6pts]

Exercise 3. Consider the subspace $U = \{(x, 2024x, y) \mid x, y \in \mathbb{R}\} \subseteq \mathbb{R}^3$. Find a subspace $W \subseteq \mathbb{R}^3$ such that

$$\mathbb{R}^3 = U \oplus W.$$

Hint. Make sure you prove that your set W is a subspace and $U + W = \mathbb{R}^3$.

Exercise 4. Let $v_1, \ldots, v_n$ and $w_1, \ldots, w_m$ be vectors in vector space V. Prove that

$$\operatorname{span}(v_1, \ldots, v_n) = \operatorname{span}(w_1, \ldots, w_m)$$

if and only if $v_i \in \operatorname{span}(w_1, \ldots, w_m)$ for all $i = 1, \ldots, n$ and $w_j \in \operatorname{span}(v_1, \ldots, v_n)$ for all $j = 1, \ldots, m$.

Remark. This result is very useful since it gives us a method to prove two sets of elements produce the same spanning set.

Additional Suggested Problems. [Not graded]

Problems 1.C # 10, 19, 20, 2.A # 2, 3, 7