Math 2R03 (Theory of Linear Algebra)
Due: October 11, 2024

Homework Assignment 4

All of the questions from Part A will be graded. One of the questions from Part B will be graded in detail, while the others will be marked for completion. Assignments will be submitted via *Crowdmark*. You will be graded on your solution *and* how well you write your proof.

Part A. [Short Questions; 4pts]

Exercise 1. Let $V = \mathbb{R}^2$ with basis $\mathcal{B} = \{(2,5),(3,7)\}$. Let $T \in \mathcal{L}(V,V)$ and suppose that

$$T(x,y) = (2024x, 2024x + 2024y).$$

Compute $\mathcal{M}(T,\mathcal{B},\mathcal{B})$ (Remember, this means finding the matrix of T with respect to a particular basis $\mathcal{B}$)

Hint. You are allowed to use Matlab or Octave for Exercise 1 if you wish.

Exercise 2. Let $V = \mathcal{P}_{2024}(\mathbf{F})$ and $W = \mathcal{P}_{10}(\mathbf{F})$. Exlain why there is no $T \in \mathcal{L}(V,W)$ with $\dim \operatorname{null}(T) = 312$.

Part B. [Proof Questions; 6pts]

Exercise 3. Suppose that $v_1,\ldots,v_{2024} \in \mathbf{F}^{2023}$ are 2024 distinct vectors. Prove that for any vector space W and for any linear map $T \in \mathcal{L}(\mathbf{F}^{2023},W)$, the vectors $Tv_1,\ldots,Tv_{2024}$ are linear dependent in W.

Exercise 4. Let V and W be finite dimensional vector spaces (with $\dim V \neq 0$ and $\dim W \neq 0$) and $T \in \mathcal{L}(V,W)$. Suppose that there are bases $\mathcal{B} = \{v_1,\ldots,v_n\}$ of V and $\mathcal{C} = \{w_1,\ldots,w_m\}$ for W such that $\mathcal{M}(T,\mathcal{B},\mathcal{C})$ with respect to these bases contains only the values 2024, i.e.,

$$\mathcal{M}(T,\mathcal{B},\mathcal{C}) = \begin{bmatrix} 2024 & \cdots & 2024 \\ \vdots & & \vdots \\ 2024 & \cdots & 2024 \end{bmatrix}.$$

Prove that $\dim \operatorname{Null}(T) = \dim V - 1$.

Additional Suggested Problems. [Not graded]

Problems 3.A #2, 4, 7, 3.B #2, 6, 9, 3.C #2, 5, 6, 3.D #2, 3, 5