Math 2R03 Midterm 2 Info Sheet

The purpose of this handout is to help you study by listing the concepts, definitions, and results you will need to know for the midterm.

Midterm Information. The midterm will be on **Friday, November 15, 2024**. The midterm will be 50 minutes long, held during the regular class time (10:30-11:20). To find the location of your midterm, please consult Avenue-to-Learn. Please remember to bring your **Student Card** with you.

The midterm will have 7 questions (3 questions worth two points each, 1 question worth 4 points, and 3 questions worth five points each). The midterm is out of 25 points. You may also use the standard McMaster calculator.

Material Covered. The midterm will cover the material we discussed in class in Lectures 9-21 (Chapters 3.A-3.D, 4, 5.A-5.D). Below is a breakdown of what you will need to know from each section. Note that when you are learning definitions, it is good to know an example of that definition, and an example of an object that does not satisfy the definition.

Section 3.A Know the definition of a linear map, and the notation $\mathcal{L}(V, W)$. Know how to check if a function is a linear map. Know Theorems 3.4. Know the algebraic operations on $\mathcal{L}(V, W)$ that make $\mathcal{L}(V, W)$ a vector space. Also know Definition 3.7 and Theorem 3.10.

Section 3.B Know what the definitions of the null space and range. Know then an operator is injective or surjective. Know Theorems 3.15 and 3.18. Know the Fundamental Theorem of Linear Maps (Theorem 3.21). Know Theorems 3.22 and 3.23. You won't be tested on Theorems 3.26, 3.28.

Section 3.C Know the definition of a matrix of linear map, and be able to compute this matrix from a given basis. Know how to add matrices and multiple matrices by scalars. Know what this means for the associated matrix (Theorems 3.35 and 3.38). Know Theorem 3.40. Know the definition of matrix multiplication and Theorem 3.43. You won't be tested on the material on pages 75-79.

Section 3.D Know what it means for a map to be invertible, and what it means for a linear map to have an inverse. Know Theorem 3.63 and 3.65 which gives an equivalent way to check invertibility. Know what it means for two vector spaces to be isomorphic, and know what an isomorphism is. Know Theorem 3.70, 3.71, and 3.72. Know what we mean by the matrix of a vector (Definition 3.73), and Theorem 3.76. You won't be tested on the section "Change of Basis".

Chapter 4 For Chapter 4, you only need to know what a root of a polynomial is, how to factorize a polynomial over $\mathbb{C}$ (Theorem 4.14) and how to factorize a polynomial over $\mathbb{R}$ (Theorem 4.17). You should also know the Fundamental Theorem of Algebra. You will not be tested on the proofs of these results.

Section 5.A Know what an invariant subspace is. Know what an eigenvalue and and eigenvector of a linear operator are. Know the equivalent conditions for λ to be an eigenvalue (Theorem 5.7). Know Theorem 5.11 and 5.12. Know what it means for a polynomial to be applied to operators (Definition 5.14).

Section 5.B Know Theorem 5.19 (pay attention to the hypothesis that $F = \mathbb{C}$). Now what we mean by a monic polynomial, Theorem 5.22, and the definition of a minimal polynomial. Know Theorem 5.27, 5.29, 5.31, and 5.32. You do not need to know the material in "Eigenvalues on Odd-Dimensional Real Vector Spaces."
Section 5.C Know what we mean by $\mathcal{M}(T)$ for an operator $T \in \mathcal{L}(V)$ (definition 5.35). Know what it is meant by an upper-triangular and diagonal matrix. Know the conditions for being an upper-triangular matrix, and know Theorems 5.40, 5.41, 5.44. and 5.47.

Section 5.D Know what we mean by the eigenspace of an operator T with eigenvalue λ. Know what it means for an operator to be diagonalizable, and know Theorem 5.55 which gives equivalent conditions for an operator to be diagonalizable. Know Theorem 5.54 and 5.58. You don't need to know the material on pages 167-171.

If you have questions, please feel free to email me. Good luck!