Math 4GR3 Midterm Info Sheet

The purpose of this handout is to help you study by listing the concepts, definitions, and results you will need to know for the midterm.

Midterm Information. The midterm will be on Thursday, February 15, 2024 at 1:30PM. The midterm will take place in

BSB B103 (our regular classroom on Thursdays)

and will be 50 minutes long. There are two parts to the midterm. The first part consists of computational type problems and definition type problems. There are six of questions of this type. In the second part, four questions are given, of which you must do three.

You will *not* be allowed to bring in any notes or use the text book, but you may use the standard McMaster calculator. Please bring your **Student Card**.

Material Covered. The midterm will cover the material we discussed in class about Chapters 12-15. Below is a breakdown of what you will need to know from each section. Note that when you are learning definitions, it is good to know an example of that definition, and an example of an object that does not satisfy the definition. Note that I am assuming you know some of main results from 3GR3 (e.g., Lagrange's Theorem, First Isomorphism Theorem, the definition of a normal subgroup).

Section 12.1 and 12.2. You won't be tested explicitly on this section, but reading this section is good review since it gives good examples of groups in linear algebra.

Section 13.1. Know what we mean by a generator of a group and what it means for a group to finitely generated. Know the statement of the Fundamental Theorem of Finite Abelian Groups, and know how to use to find all the non-isomorphic finite abelian groups of a particular n. Know what is meant by a p-group and its properties (e.g. Lemma 13.7). Know Lemma 13.6.

Section 13.2. Know what be mean by a subnormal series, a normal series, a composition series, a principal series. Know what it means for two (sub)normal series to be isomorphic. Know the statement of the Jordan-Hölder Theorem. Know who to construct a composition series of $\mathbb{Z}_n$. Know what it means for a group to be solvable.

Section 14.1. Know what we mean by a group action, and know review the examples of group actions given in class. Know how a group action on a set X defines an equivalence relation on X. Know what an orbit is. Know what the fixed point set of g is, and know what the stabilizer subgroup G_x of x is (in particular, know Proposition 14.10). Know Theorem 14.11.

Section 14.2. Know what we mean by X_G, the fixed points of X. Know what the center of a group is. Know what the centralizer subgroups are. Know what we mean by the class equation of a group. Be able to compute the class equations for small groups. Know some applications of the class equations (e.g., Theorems 14.15, Corollary 14.16).

Section 14.3. You will not be tested on Burnside's Counting Theorem.

Section 15.1. Know Cauchy's theorem (Theorem 15.1) and Corollary 15.2. Know the statement of the First Sylow Theorem and its proof. Know what we mean by a Sylow p-subgroup. Know what we mean by the normalizer of H in G. Know Lemma 15.5. Know the statements of Second and Third Sylow Theorems.

Section 15.2. Be able to use the Sylow Theorems to count the number of Sylow p-subgroups in a group G. Know Theorem 15.10. Of special importance is Exercise 11 of Section 15.3 (in fact, as shown in class, we showed that a Sylow p-subgroup is normal if and only if G has only one Sylow p-subgroup.

If you have questions, please feel free to email me. Good luck!