Challenge Exercise 1
MATH 2255 – 2005
Due Date: Sept 28, 2005

These challenge exercises ask you questions about material covered in class, but at a greater depth. You are not required to do this exercise; it is intended as extra work. However, you will receive extra credit if you complete the solutions correctly.

When handing this assignment in, please clearly label your work as a Challenge Exercise. Make sure to include your name. For those of you in Math 2231 (Ring Theory), you are encouraged to write you solutions as a formal proof.

Problem 1. [**5pts**] We introduce the following definition.

Definition. An $m \times n$ matrix M is called *row-reduced* if

(i) the first non-zero entry in each non-zero row of M is equal to 1.
(ii) each column of M which contains the leading non-zero entry of some row has all its other entries 0.

Note that this similar to the row-reduced echelon form we talked about class, but we do not require the echelon part of the definition.

Let

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

be a 2×2 matrix with entries in the complex numbers. Suppose that A is row-reduced and also that $a + b + c + d = 0$. Prove that there are exactly three such matrices.

Problem 2. [**5pts**] Prove that the following two matrices are *not* row-equivalent:

$$\begin{bmatrix} 2 & 0 & 0 \\ a & -1 & 0 \\ b & c & 3 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{bmatrix}$$

Here, a, b and c are any real number.