Challenge Exercise 2
MATH 2255 – 2005
Due Date: Oct 12, 2005

These challenge exercises ask you questions about material covered in class, but at a greater depth. You are not required to do this exercise; it is intended as extra work. However, you will receive extra credit if you complete the solutions correctly.

When handing this assignment in, please clearly label your work as a Challenge Exercise. Make sure to include your name. For those of you in Math 2231 (Ring Theory), you are encouraged to write you solutions as a formal proof.

Problem 1. [**10pts**] For a certain collection of vectors $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4, \mathbf{a}_5, \mathbf{a}_6$ in $\mathbb{R}^5$ it is known that $\mathbf{a}_1, \mathbf{a}_4$ and $\mathbf{a}_6$ are linear independent, and that

$$\mathbf{a}_4 = 4\mathbf{a}_1 + 2\mathbf{a}_2 + \mathbf{a}_3 \text{ and } \mathbf{a}_5 = 2\mathbf{a}_1 - 3\mathbf{a}_2 + \mathbf{a}_6.$$

It is known that if E is the 5×6 row reduced echelon form of the matrix $A = [\mathbf{a}_1 \; \mathbf{a}_2 \; \mathbf{a}_3 \; \mathbf{a}_4 \; \mathbf{a}_5 \; \mathbf{a}_6]$, then E has the form

$$\begin{bmatrix} \star & \star & \star & -4 & \star & \star \\ \star & 0 & 1 & \star & \star & \star \\ \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star \end{bmatrix}$$

where $\star$ is some integer. Use the given information to fill in the rest of the matrix.

Hint. Use the following fact:

Fact. *Let* $A = [\mathbf{a}_1 \; \mathbf{a}_2 \; \cdots \; \mathbf{a}_n]$ *be an* $m \times n$ *matrix, and let* $E = [\mathbf{e}_1 \; \mathbf{e}_2 \; \cdots \; \mathbf{e}_n]$ *be the unique row reduced echelon matrix that is row equivalent to* A. *Then*

$$c_1\mathbf{a}_1 + c_2\mathbf{a}_2 + \cdots + c_n\mathbf{a}_n = \mathbf{0}$$

is a linear dependent relation among the columns of A *if and only if*

$$c_1\mathbf{e}_1 + c_2\mathbf{e}_2 + \cdots + c_n\mathbf{e}_n = \mathbf{0}$$

is also a linear dependent relation among the columns of E.