These challenge exercises ask you questions about material covered in class, but at a greater depth. You are not required to do this exercise; it is intended as extra work. However, you will receive extra credit if you complete the solutions correctly.

When handing this assignment in, please clearly label your work as a Challenge Exercise. Make sure to include your name. For those of you in Math 2231 (Ring Theory), you are encouraged to write you solutions as a formal proof.

Problem 1. [**5pts**] Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation. Prove that if $T(\mathbf{x}) = (cI_n)(\mathbf{x})$ for all $\mathbf{x} \in \mathbb{R}^n$, then $T \circ S = S \circ T$ for all linear transformations $S : \mathbb{R}^n \to \mathbb{R}^n$. (Note: I_n is the identity matrix and c is a scalar.)

Problem 2. [**5pts**] Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ and $U : \mathbb{R}^2 \to \mathbb{R}^3$ be two linear transformations. Explain why the linear transformation $U \circ T$ is not invertible.