
Challenge Exercise 4
MATH 2255 – 2005
Due Date: Nov. 9, 2005

These challenge exercises ask you questions about material covered in class, but at a greater depth. You are not required to do this exercise; it is intended as extra work. However, you will receive extra credit if you complete the solutions correctly.

When handing this assignment in, please clearly label your work as a Challenge Exercise. Make sure to include your name. For those of you in Math 2231 (Ring Theory), you are encouraged to write your solutions as a formal proof.

Problem 1. We introduce the following definition.

Definition. A square matrix A is said to **nilpotent of index k** if $A \neq 0$, $A^2 \neq 0, \dots, A^{k-1} \neq 0$, but $A^k = 0$.

Answer the following questions:

- (i) [2pts] Find 4×4 nilpotent matrices of indices 2 and 4.
- (ii) [2pts] What is the determinant of a nilpotent matrix? Justify your answer.
- (iii) [3pts] If A is a $n \times n$ nilpotent matrix of index k , show that $I_n - A$ is an invertible matrix.

Hint: Adapt the following identity:

$$(1 - x)(1 + x + x^2 + x^3 + \dots + x^{k-1}) = (1 - x^k)$$

- (iv) [3pts] If A is nilpotent, what can you say about A^T , the transpose? Justify your answer.