Challenge Exercise 5
MATH 2255 – 2005
Due Date: Nov. 23, 2005

These challenge exercises ask you questions about material covered in class, but at a greater depth. You are not required to do this exercise; it is intended as extra work. However, you will receive extra credit if you complete the solutions correctly.

When handing this assignment in, please clearly label your work as a Challenge Exercise. Make sure to include your name. For those of you in Math 2231 (Ring Theory), you are encouraged to write you solutions as a formal proof.

Problem 1. We introduce the following definition:

Definition. An $n \times n$ matrix M is called a **magic square** if the sum of the entries is the same in each row, each column, and both diagonals.

Here's an example of a magic square:

$$\begin{bmatrix} 16 & 3 & 2 & 13 \\ 5 & 10 & 11 & 8 \\ 9 & 6 & 7 & 12 \\ 4 & 15 & 14 & 1 \end{bmatrix}$$

Each row, column and diagonal sums to 34.

Definition. If M is a magic square, the common sum is called the **weight** of M, denoted $\mathrm{wt}(M)$. If M is a $n \times n$ magic square that contains each of the entries $1, 2, \ldots, n^2$ exactly once, then M is called a **classical magic square**. (The example above is a classical magic square.)

Answer the following questions:

(*i*) [**2pts**] Find a classical 3×3 magic square.

(*ii*) [**3pts**] Show that if M is a classical $n \times n$ magic square, then

$$\mathrm{wt}(M) = \frac{n(n^2+1)}{2}.$$

(*iii*) [**5pts**] Let Mag_n denote the set of all $n \times n$ matrices that are magic squares. So, Mag_n is a subset of M_{nn}, the set of all $n \times n$ matrices. Show that Mag_n is a subspace of M_{nn}. (You may use the fact that M_{nn} is a vector space without proof.)