NAME: _____

STUDENT NUMBER: _____

MATH 1281 - Final Exam

Lakehead University

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Instructions: Answer all questions in the space provided. If you need more room, answer on the back of the page. Where appropriate, you must provide clear explanations.

You are *not* allowed to use a calculator. If a question involves a calculation you may leave it in an unexpanded form, e.g., you can write 5^4 instead of 625.

If doubt exists as to the interpretation of any question, you are urged to submit with the answer paper a clear statement of any assumptions made.

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Total	100	

Chapter 1 – Logic, Sets, and Functions

1. [**2pts**] Let p and q be propositions. Write out the truth table for the statement $(p \to q) \wedge p$.

2. [**4pts**] What are the converse and contrapositive of the statement "If I go to the store, then I will buy milk"?

3. [**2pts**] Let A, B, and C be sets. Prove or disprove that

$$A - (B \cap C) = (A - B) \cup (A - C).$$

Chapter 2 – Algorithms, Integers, and Matrices

4. [**2pts**] Use the Euclidean algorithm to find $\gcd(203, 101)$. Write out all your steps.

5. [**4pts**] Let m be a positive integer, and let a, b, and c be integers. Prove that if $a \equiv b \pmod{m}$, then $a - c \equiv b - c \pmod{m}$.

Chapter 3 – Methods of Proof

6. [**2pts**] Find the value of $\sum_{j=1}^{1234}(-1)^j$

7. [**4pts**] Use the Principle of Mathematical Induction to prove that $2|(n^2+3n)$ for all $n \geq 1$.

Chapter 4 – Counting

8. [**8pts**] Consider the set $S = \{1, \ldots, 10\}$.

   (*i*) Find the number of subsets of S. (Recall that the empty set is also considered a subset of S.)

   (*ii*) Find the number of subsets of S that contain the number 5.

   (*iii*) Find the number of subsets of S that does not contain either 5 or 6.

   (*iv*) Find the number of subsets of S that contain no odd numbers.

Chapter 5 – Discrete Probability

9. [**2pts**] A group of ten women and ten men are in a room. A committee of four is chosen at random. Find the probability that the committee consists only of women.

10. [**4pts**] Find and correct the error in the solution to the following problem:

Problem: You flip two coins and want to find the probability that both coins show heads.

Solution: There are three possible outcomes: 2 heads, 2 tails, 1 head and 1 tail. Since a "success" is one of these three outcomes, $p(\text{both head}) = 1/3$.

Chapter 6 – Advance Counting Techniques

11. [**2pts**] Find a closed form for the generating function for the sequence

$$2, 0, 0, 2, 0, 0, 2, 0, 0, 2, 0, 0, 2, 0, \ldots$$

(Hint: $\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + x^3 + \cdots$)

12. [**4pts**] Solve the recurrence relation

$$a_n = 6a_{n-1} - 9a_{n-2}$$

when $a_0 = 1$ and $a_1 = 2$.

Chapter 7 – Relations

13. [**4 pts**] Let $A = \{w, x, y, z\}$. Define a relation R on A such that

(*i*) R is a partial ordering.

(*ii*) R is neither symmetric nor antisymmetric.

You may express your relation as a zero-one matrix.

14. [**4pts**] Order the following tuples of $\mathbb{N}^5$ in lexicographical order:

$$(1, 0, 4, 3, 2), (2, 23, 0, 4, 2), (0, 1, 0, 754, 34), (1, 0, 14, 4, 1), (6, 0, 1, 4, 1).$$

Chapter 8 – Graphs

15. [**4pts**] Give an example of a graph G with a cut edge and cut vertex. Label the cut edge and cut vertex in your graph.

16. [**4pts**]

   (*i*) Explain why it is not possible to draw simple connected graph with 8 vertices where the degrees of the vertices are 1, 1, 1, 2, 3, 4, 5, and 7.

   (*ii*) Show that it possible (by giving an a example) to draw a multi-graph with 8 vertices where the degrees of the vertices are 1, 1, 1, 2, 3, 4, 5, and 7.

17. [**4pts**] Draw the graph represented by the given adjacency matrix:

$$(i)\quad \begin{pmatrix} 0 & 1 & 2 & 0 \\ 2 & 1 & 1 & 0 \\ 2 & 1 & 0 & 3 \\ 1 & 1 & 0 & 0 \end{pmatrix} \qquad (ii)\quad \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

18. [**4pts**] Determine the value(s) of n for which the complete graph K_n has

(*a*) an Euler circuit.

(*b*) a Hamilton circuit.

19. [**6pts**] Let G be a planar graph. The *region colouring* of G is an assignment of colours to the regions of G so that no two adjacent regions have the same colour. The *region-chromatic number* is the minimum number of colours need in a region colouring of G. Find the region-chromatic number for the following graphs

(*i*) C_n for any $n \geq 3$.

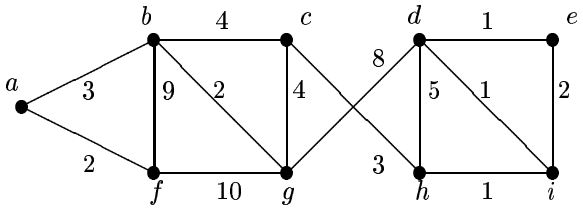
(*ii*) K_4

(*iii*) T where T is any tree.

Justify your answers.

Chapter 9 – Trees

20. [**10 pts**] For the questions below, use the following graph:



(*i*) Adapt Kruskal's Algorithm to find a **maximal spanning tree** in the weighted graph. (Recall that Kruskal's algorithm as explained in class finds a minimal spanning tree.) Write out the steps in the algorithm, that is, list the order in which you picked the edges for your spanning tree.

(*ii*) Find the length of the shortest path from a to i (you may do this by inspection).

(*iii*) What is the chromatic number of the above graph? Justify your answer. (Note that the weighting of the graph does not matter)

21. [**4pts**] Form a binary search tree from the words of the sentence THIS TEST IS NOT SO DIFFICULT.

Chapter 10 – Boolean Algebras

22. [**2 pts**] Verify the identity $\overline{x} \cdot \overline{y} = \overline{x + y}$.

23. [**4 pts**] Use a Karnough map to simplify the Boolean function $xyz + x\overline{y}z + x\overline{y}\ \overline{z} + \overline{x}\ \overline{y}z$.

24. [**8pts**] Three people are on a committee. Construct a circuit using inverters, AND gates, and OR gates that gives an output of 1 if the three people on a committee do not all vote the same.

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