Challenge Exercise 1
MATH 2275 – Winter 2006
Due Date: Feb. 1, 2006

These challenge exercises ask you questions about material covered in class, but at a greater depth. You are not required to do this exercise; it is intended as extra work. However, you will receive extra credit if you complete the solutions correctly.

When handing this assignment in, please clearly label your work as a Challenge Exercise. Make sure to include your name. For those of you in Math 2231/2233 (Ring/Group Theory), you are encouraged to write you solutions as a formal proof.

Problem Let $p(x)$ be the polynomial

$$p(x) = x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0.$$

The **companion matrix** of $p(x)$ is the $n \times n$ matrix

$$C(p) = \begin{bmatrix} -a_{n-1} & -a_{n-2} & \cdots & -a_1 & -a_0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

(*a*) [**2 pts**] Find the companion matrix of $p(x) = x^2 - 7x + 10$. Then find the characteristic polynomial of the matrix $C(p)$.

(*b*) [**2 pts**] Find the companion matrix of $p(x) = x^3 + 3x^2 - 4x + 12$. Then find the characteristic polynomial of the matrix $C(p)$.

(*c*) [**2 pts**] Show that the companion matrix $C(p)$ of $p(x) = x^2 + ax + b$ has characteristic polynomial $\lambda^2 + a\lambda + b$.

(*d*) [**2 pts**] Show that if λ is an eigenvalue of the companion matrix $C(p)$ from part (*c*), then $\begin{bmatrix} \lambda \\ 1 \end{bmatrix}$ is an eigenvector of $C(p)$ corresponding to λ.

(*e*) [**2 pts**] Make a conjecture about the characteristic polynomial of the matrix $C(p)$ if $p(x) = x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots a_1 x + a_0$.