Challenge Exercise 2
MATH 2275 – Winter 2006
Due Date: Feb. 15, 2006

These challenge exercises ask you questions about material covered in class, but at a greater depth. You are not required to do this exercise; it is intended as extra work. However, you will receive extra credit if you complete the solutions correctly.

When handing this assignment in, please clearly label your work as a Challenge Exercise. Make sure to include your name. For those of you in Math 2231/2233 (Ring/Group Theory), you are encouraged to write you solutions as a formal proof.

Problem. In class we defined $||\mathbf{v}||$, the notion of a *norm* of vector $\mathbf{v}$ (See Section 6.1). We can also define a notion of norm for a matrix.

Precisely, let A be an $n \times n$ matrix with $A = [a_{ij}]$. Define the norm of A to be

$$||A|| = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}^2}$$

Using this definition, answer the following questions.

(a) [**2pts**] Find $||A||$ when

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -5 & 7 \\ 4 & 8 & -1 \end{bmatrix}$$

(b) [**2pts**] Prove that $||A|| = 0$ if and only if $A = O$, the zero matrix.

(c) [**4pts**] Let A be any 2×2 matrix, and show that for any vector $\mathbf{v} \in \mathbb{R}^2$, we have

$$||A\mathbf{v}|| \leq ||A|| \; ||\mathbf{v}||$$

Read this carefully: since $A\mathbf{v}$ and $\mathbf{v}$ are vectors, by $||A\mathbf{v}||$ and $||\mathbf{v}||$ we mean the norm of the vectors as defined in class.

(d) [**2pts**] Let λ be any eigenvalue of a 2×2 matrix A. Show that $||A|| \geq |\lambda|$.